Matter in external electric fields
Gauss’s law

- Flux of electric field produced by distribution of charges.

- The electric flux through any closed surface is equal to the net charge enclosed by the surface divided by permittivity of the medium.
Flux of uniform electric field

\[ \Phi_E = E S \cos \alpha \]

\[ \vec{E} \cdot \vec{S} = E S \cos \alpha = \Phi_E \]

Gauss’s law
\[ d\Phi_E = \vec{E} \cdot d\vec{S} \]

flux of electric field

\[ \Phi_E = \int_S \vec{E} \cdot d\vec{S} \]
Gauss’s law

\[ \Phi = \frac{Q}{\varepsilon_0} \]

- The electric flux through any closed surface is equal to the net charge enclosed by the surface divided by permittivity of the medium.
Electric field of a plate

- application of Gauss’s law to a cylinder

\[ 2ES = \frac{Q}{\varepsilon_0} \]

- surface density of charge

\[ \sigma = \frac{Q}{S} \]

- uniform electric field

\[ E = \frac{\sigma}{2\varepsilon_0} \]
Electric field of a plate capacitor

- net field outside of capacitor is zero
- net field inside of capacitor is:

\[ E = E_+ + E_- \]

\[ E = \frac{\sigma}{\varepsilon_0} \]
- flux of electric field: \[ \Phi = \vec{E} \cdot \vec{S} \]

- Gauss’s law

\[ \Phi = \frac{Q}{\varepsilon_0} \]

- charge surface density: \[ \sigma = \frac{Q}{S} \]

- uniform electric field of plate capacitor

\[ E = \frac{\sigma}{\varepsilon_0} \]
Biological tissue in external electric field?

Polarization of matter in electric field
Conductors in external electric field

- metals
- electrolytes
- free electrons and ions
- The strength of external uniform electric field $E_0$

No net electric field:

$$E_{\text{conductor}} = -E_0$$
Dielectrics in electric field

- No free charges

- Permanent and induced electric dipoles orient themselves in an external electric field

- Induced electric field of dielectric: $E_d$

- The net electric field: $E = E_0 - E_d$

[https://www.doitpoms.ac.uk/tlplib/dielectrics/capacitors.php](https://www.doitpoms.ac.uk/tlplib/dielectrics/capacitors.php)
- pair of forces rotates the dipole in an electric field

- torque of the pair of forces

\[ \vec{\tau} = \vec{p} \times \vec{E} \]

http://physics.mef.hr/Predavanja/tvariupolju/index.html
Polarization of dielectric

- Charges on dielectrics surface in external electric field

- The net electric field:
  \[ E = E_0 - E_d \]

- By Gauss’s law:
  \[ E = \frac{\sigma_0}{\varepsilon_0} - \frac{\sigma_d}{\varepsilon_0} = \frac{1}{\varepsilon_0} \left( \sigma_0 - \sigma_d \right) \]
Electric field of capacitor with dielectric

$E = E_0 / \varepsilon_r$

Relative permittivity defines relation between electric field of a capacitor with and without dielectric.

$$E_d = E_0 - E = \varepsilon_r E - E$$

$$E_d = E(\varepsilon_r - 1)$$

Dielectric susceptibility
- vector of polarization – measure of polarization

\[ \mathbf{P} = \frac{\sum_i \mathbf{p}_i}{V} \]

- charge surface density

\[ P = \sigma_d \]

\[ P = \varepsilon_0 E_d = \varepsilon_0 \chi E \]
Relative permittivity

\[ \varepsilon_r = \frac{E_0}{E} \geq 1 \]

- high relative permittivity – electric dipoles are oriented in the electric field better

<table>
<thead>
<tr>
<th>matter</th>
<th>( \varepsilon_r )</th>
</tr>
</thead>
<tbody>
<tr>
<td>organic solvents</td>
<td>2 - 5</td>
</tr>
<tr>
<td>fat tissue</td>
<td>12</td>
</tr>
<tr>
<td>glass</td>
<td>6 - 10</td>
</tr>
<tr>
<td>cow’s milk</td>
<td>66</td>
</tr>
<tr>
<td>water</td>
<td>81</td>
</tr>
<tr>
<td>white tissue - brain</td>
<td>90</td>
</tr>
<tr>
<td>gray tissue - brain</td>
<td>85</td>
</tr>
</tbody>
</table>

- Measure of the polarization efficiency – relaxation time \( \tau \)
Matter in an electric field

- **Conductors**
  \[ E = 0 \]

- **Dielectrics**
  \[ E = E_0 - E_d \]

- **Polarization**
  \[ \sigma_d = \varepsilon_0 E_d \]

- **Relative permittivity**
  \[ \varepsilon_r = E_0 / E \]

- **Dielectric susceptibility**
  \[ \chi = E_d / E \]
Mechanisms of polarization

In a homogeneous substance:

1. DIPOLE
2. ELECTRONIC
3. IONIC

In inhomogeneous substances in additional mechanisms are:

4. PHASE
5. MEMBRANE
6. ELECTROLYTIC
Dipole polarization

- It occurs in compounds with polar molecules.

http://www.doitpoms.ac.uk/tlplib/dielectrics/dielectric_constant.php

- Polar molecules are directed toward the field.

- Directing is not perfect due to the chaotic thermal motion.

- Relaxation time: \( \tau = 10^{-10} \text{s} \)

http://physics.mef.hr/Predavanja/tvariupolju/index.html
Electronic polarization

- It occurs in compounds without polar molecules.
- Electronic cloud is deformed when they are inserted in the external field.
- Induced dipole moment: \( p \propto E_0 \)
- Relative permittivity does not depend on temperature
- Relaxation time: \( \tau = 10^{-15} \text{s} \)

http://www.doitpoms.ac.uk/tlplib/dielectrics/dielectric_constant.php
Ionic polarization

- It occurs in ionic crystals.

- The external electric field causes the spacing of ions in crystals with ionic bonds. Shifts are within the dimensions of molecules.

- Relaxation time:

\[ \tau = 10^{-13} \text{s} \]

http://www.doitpoms.ac.uk/tlplib/dielectrics/dielectric_constant.php
Phase polarization

- The appearance of the surface charge on the tissue regions with the different dielectric properties.

\[ \tau = 10^{-3} \, s \]
Membrane polarization

- When the tissue in the external electric field, the membrane polarization can be either in the direction of the field or in opposite direction.

- **It is very important for living tissue.**

- Relaxation time: \( \tau = 10^{-2} \, s \)
Before Pulse

Cell membrane

Introduce genes/drugs

Electric field induces a voltage across cell membrane

During E-field

After Pulse

Cell "heals" with gene/drug inside


Electrolyte polarization

- The whole body acts as a large induced dipole.

- Relaxation time:

  \[ \tau > 1 \text{ s} \]
Polarization of matter in alternating electric field

- Time independent electric field of capacitor
  \[ \varepsilon_r = \text{konst.} \]

- Alternating electric field of capacitor
  \[ \varepsilon_r(f) \]

\[ E = E_0 \sin \omega t \]

\[ \tau \leq \frac{T}{2} \]

\[ \omega = 2\pi f \quad T = \frac{1}{f} \]
Polarization of matter in an alternating electric field

\[ \tau < 10^{-10} \, s \quad \tau < 10^{-15} \, s \]

- relative permittivity as a function of frequency of external electric field
- Dielectric function
### Summary

- mechanism of polarization is defined by time relaxation

\[ \tau \leq \frac{T}{2} \]

<table>
<thead>
<tr>
<th>Relaxation time $\tau$ (s)</th>
<th>mechanism of polarization</th>
</tr>
</thead>
<tbody>
<tr>
<td>$10^{-15}$</td>
<td>electronic</td>
</tr>
<tr>
<td>$10^{-10}$</td>
<td>dipole</td>
</tr>
<tr>
<td>$10^{-3}$</td>
<td>phase</td>
</tr>
<tr>
<td>$10^{-2}$</td>
<td>membrane</td>
</tr>
<tr>
<td>1</td>
<td>electrolyte</td>
</tr>
</tbody>
</table>