HYDRODINAMCS

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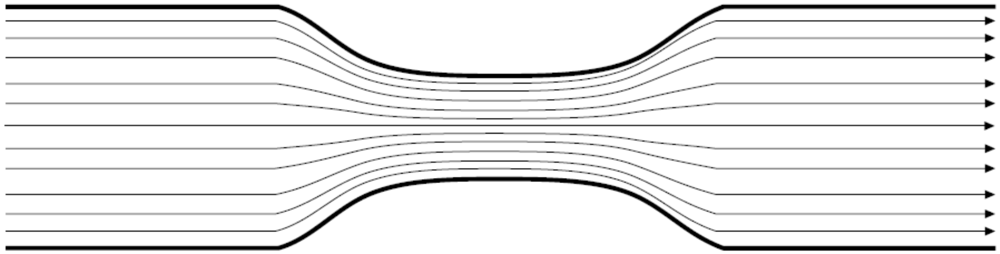
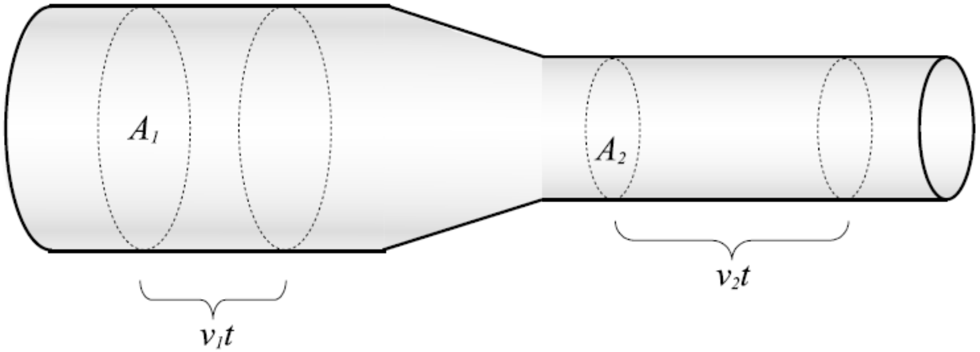
LIQUIDS

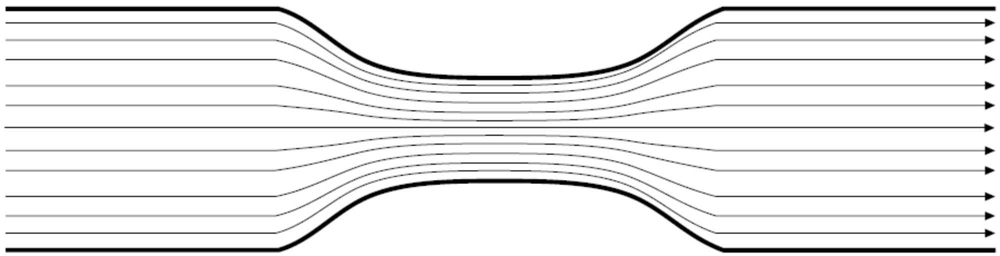


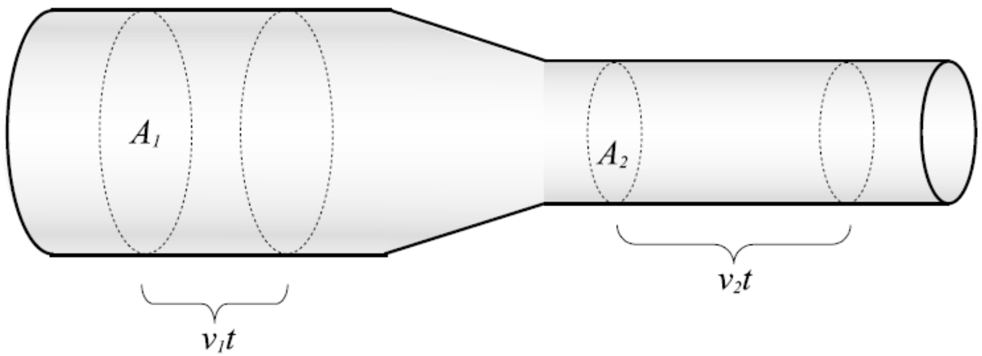


$$\frac{V}{t} = const.$$

- Constant volume flow rate is the result of the fact that all liquids are incompressible.

Model of ideal liquid

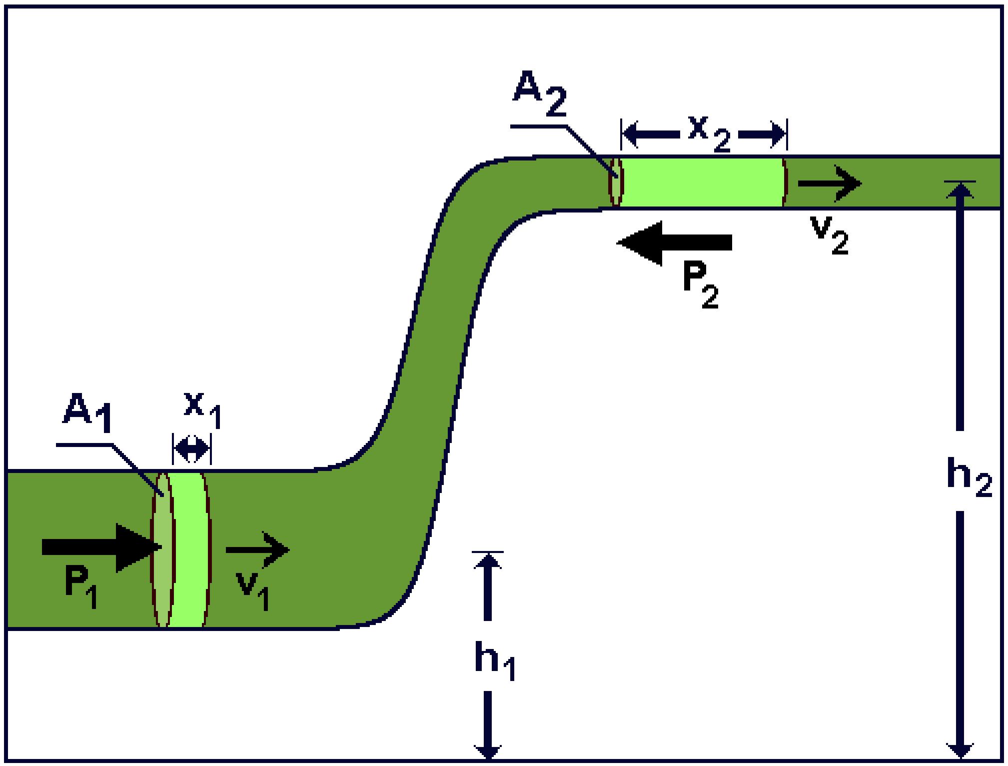




$$A_1 v_1 = A_2 v_2$$

- There is no friction between the layers of the liquid.
- There is no friction between the outer layer and the wall of the tube.
- The velocities of all layers are equal.
- There is no pressure drop along the tube.

Model of ideal liquid

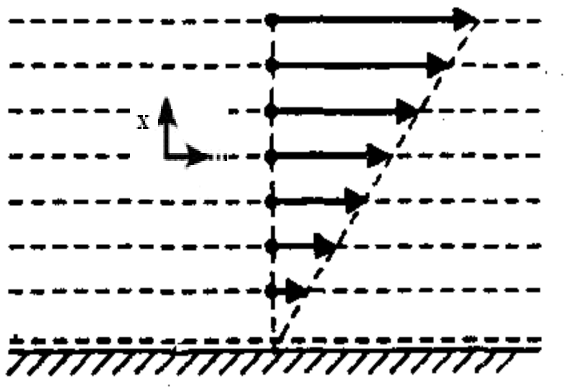


$$p_1 + \rho g h_1 + \frac{\rho v_1^2}{2} = p_2 + \rho g h_2 + \frac{\rho v_2^2}{2}$$

- Bernoulli's law of energy conservation

Model of real liquid

- There is friction between the layers of the liquid.
- Friction between the outer layer and the wall of the tube is infinite.
- The velocities of the layers decrease from the center of the tube toward its walls.
- Pressure drop along the tube is the consequence of the internal friction.



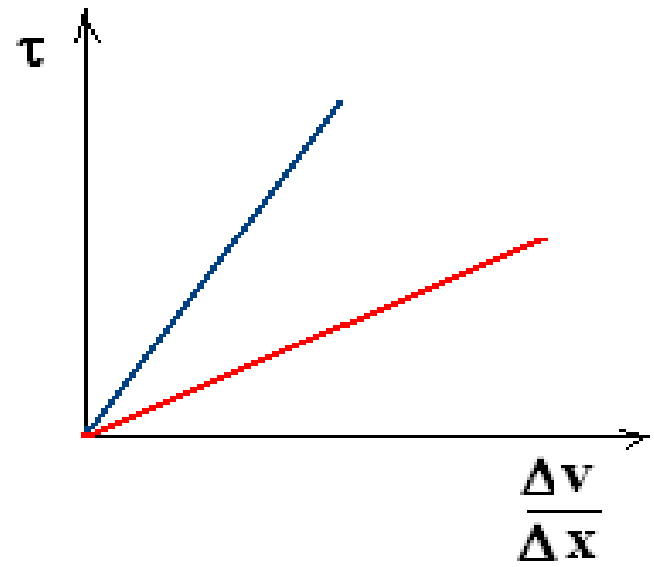
Newton's law of viscosity

$$F = \eta A \frac{\Delta v}{\Delta x}$$

velocity gradient

$$\tau = F / A = \eta \frac{\Delta v}{\Delta x}$$

tangential tension –shear stress

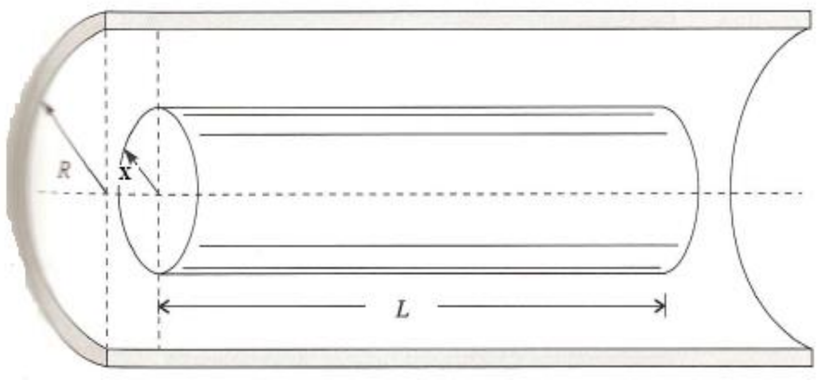


- **Newtonian liquid** – viscosity doesn't depend on the velocity gradient

$$\eta(water) = 1 \cdot 10^{-3} \text{ Pas}$$

$$\eta(blood) = 4 \cdot 10^{-3} \text{ Pas}$$

Equation of motion



- F' – force applied for the motion of layer

$$F' = (p_1 - p_2) x^2 \pi$$

- F'' – the friction force that suppresses the flow

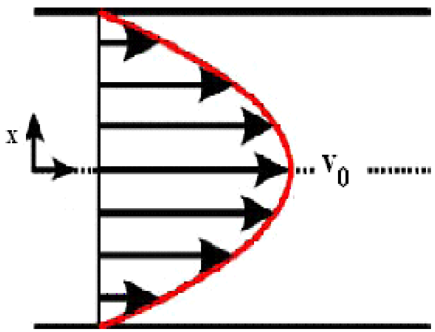
$$F'' = 2\pi\, x\, L \eta \frac{\Delta v}{\Delta x}$$

A

$$\overrightarrow{F'} + \overrightarrow{F''} = 0$$

"easy" integral calculus

$$v_0 = \frac{p_1 - p_2}{4\eta L} \cdot R^2$$



- parabolic distribution of velocities

- mean velocity: $\bar{v} = \frac{p_1 - p_2}{8\eta L} \cdot R^2$

- The volume flow rate: $\frac{V}{t} = \bar{v}\, R^2 \pi = \frac{R^4 \pi}{8\eta} \frac{p_1 - p_2}{L}$

Poiseuille's law

pressure gradient

- the Newtonian fluid viscosity is constant and does not depend on the pressure gradient → volume flow is a linear function of the pressure gradient

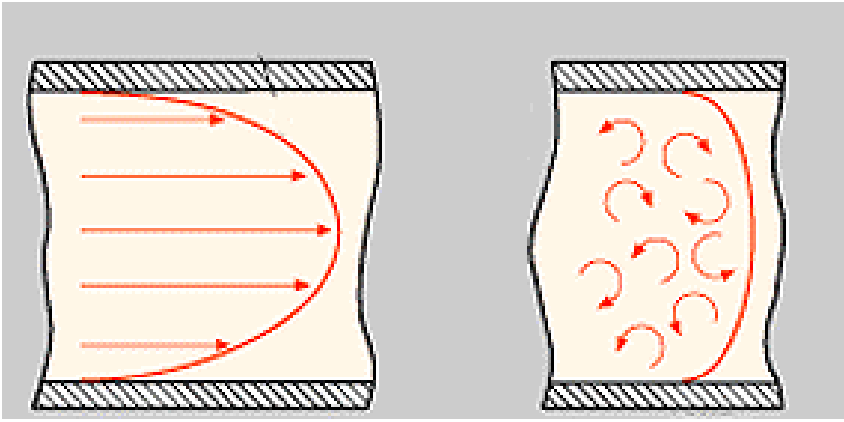
- **hidraulic resistance**: $$r = \frac{\Delta p}{V/t} \longrightarrow r = \frac{8\eta L}{R^4 \pi}$$

- analogy with electric resistance: $$R = \frac{voltage}{el.current}$$

  - depends on geometry of the tube and properties of the liquid
  - strong dependence on the tube radius

Turbulent flow

- At high speed the friction forces decrease and the layers are mixed.
- increase the flow velocity of fluid → weaker influence of attractive forces between the fluid and the pipe wall



- energy is spent on creating vortices, vibration, noise and heat
- Reynolds number quantifies the flow properties

Reynolds number

$$R_e = \frac{2\rho\,\bar{\text{v}}\,\text{R}}{\eta}$$

density of liquid

- for $R_e > 1000$ most laminar flow becomes turbulent

from the continuity equation:

$$\frac{\text{V}}{\text{t}} = A \cdot \bar{\text{v}} \longrightarrow \bar{\text{v}} \propto \frac{1}{\text{R}^2}$$

$$R_e \propto \frac{1}{R^2} R \longrightarrow R_e \propto \frac{1}{R}$$

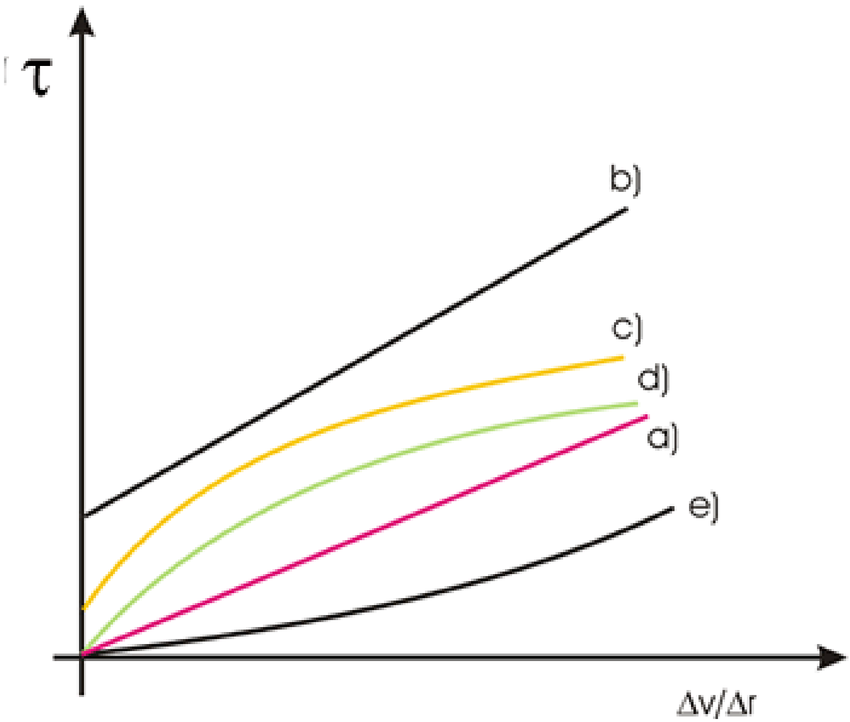
Effect of narrowing for the constant volume flow rate: In narrow parts of the tube the flow could become turbulent due to the increase of Reynolds number.

http://www.youtube.com/watch?v=47oJYhwJsJE

http://physics.mef.hr/Predavanja/hidrodinamika/index.html

Non-newtonian liquids

- Viscosity depends on the velocity and pressure gradients



Rheological models:

a – Newtonian

b – Bingham's

c – viscoplastic

d – pseudoplastic

e – dilatant liquids

http://www.youtube.com/watch?v=OBrncHJLSIU&feature=related