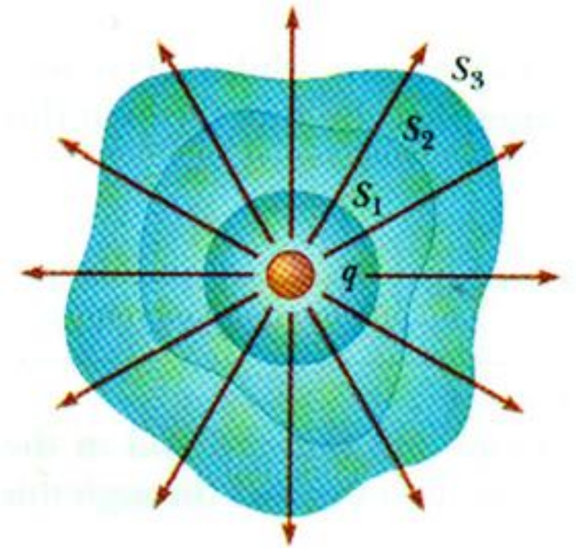
Matter in external electric fields

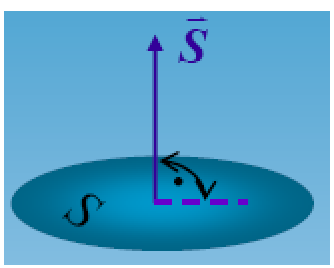
Sanja Dolanski Babić

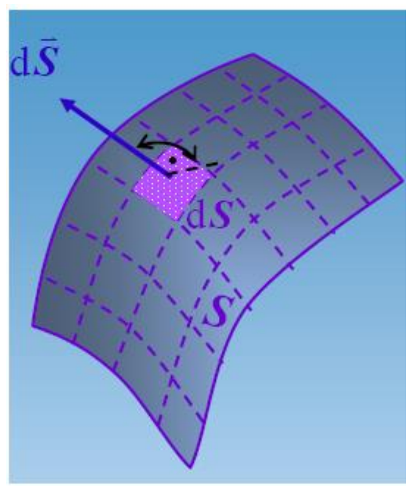
Gauss’s law

- Flux of electric field produced by distribution of charges.

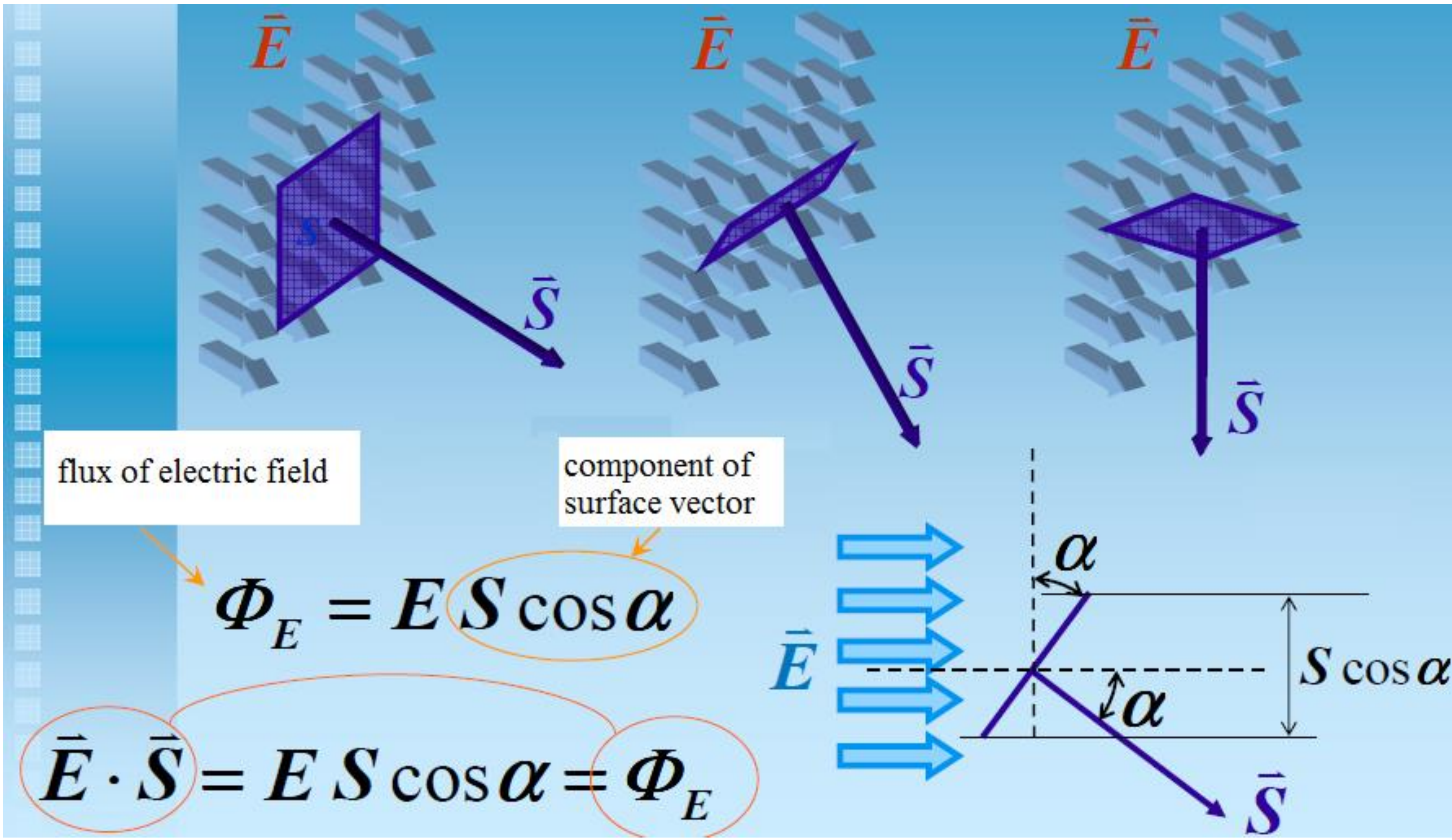
- The electric flux through any closed surface is equal to the net charge enclosed by the surface divided by permitivity of the medium.

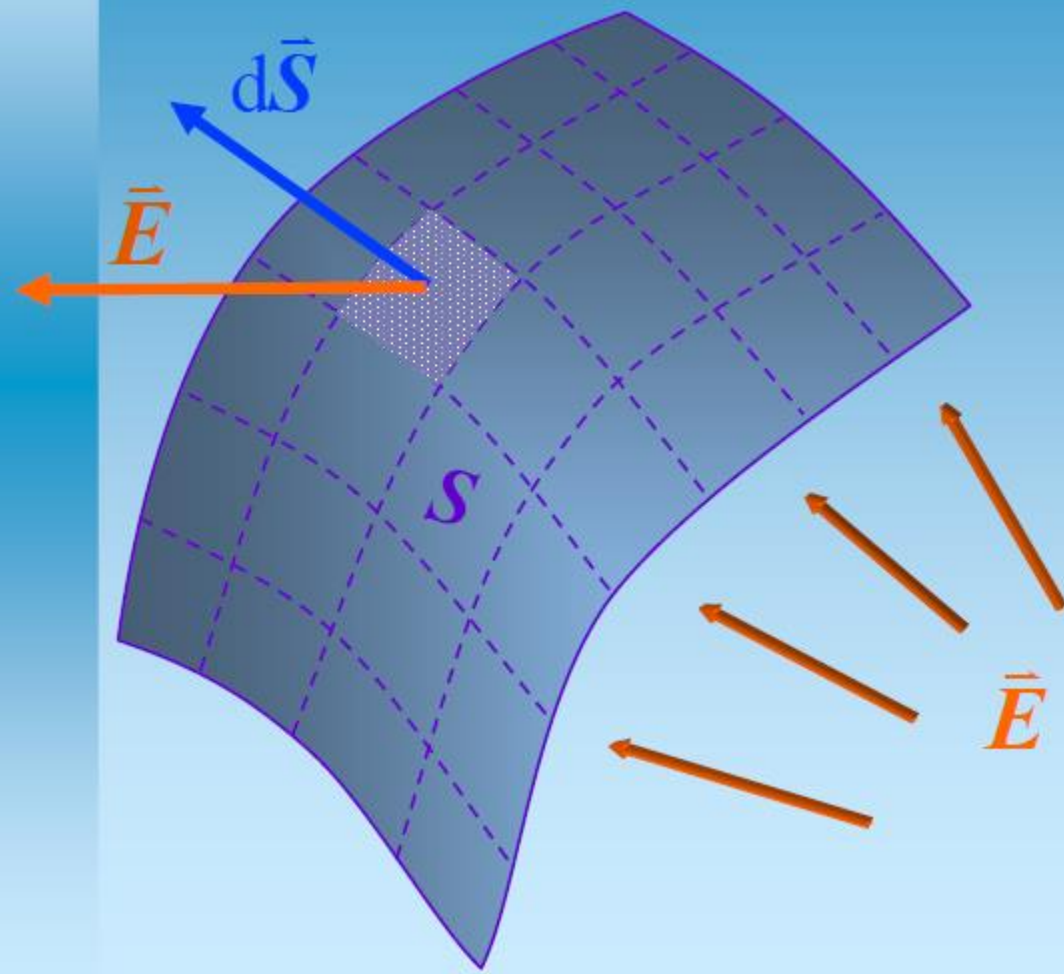






Flux of uniform electric field





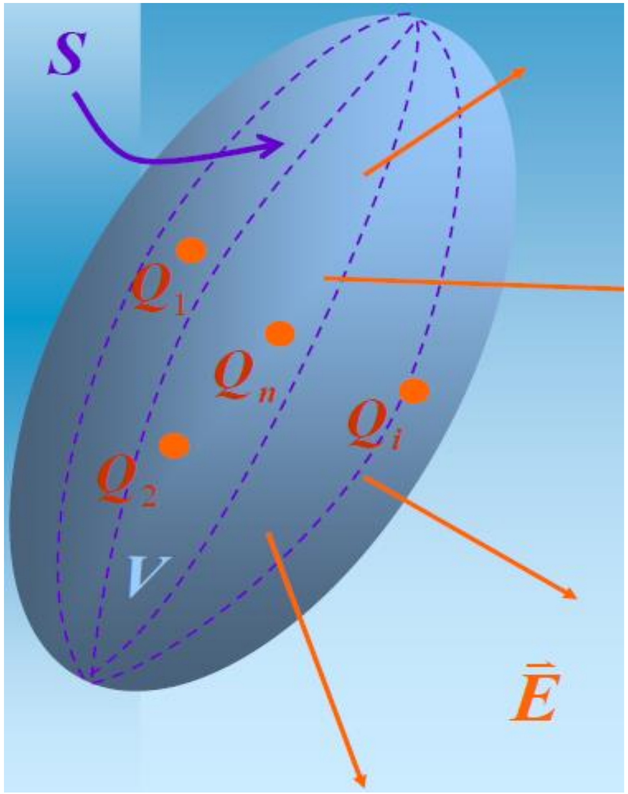
$$\mathrm{d}\Phi_E = \vec{E} \cdot \mathrm{d}\vec{S}$$

flux of electric field

$$\Phi_E = \int_S \vec{E} \cdot \mathrm{d}\vec{S}$$

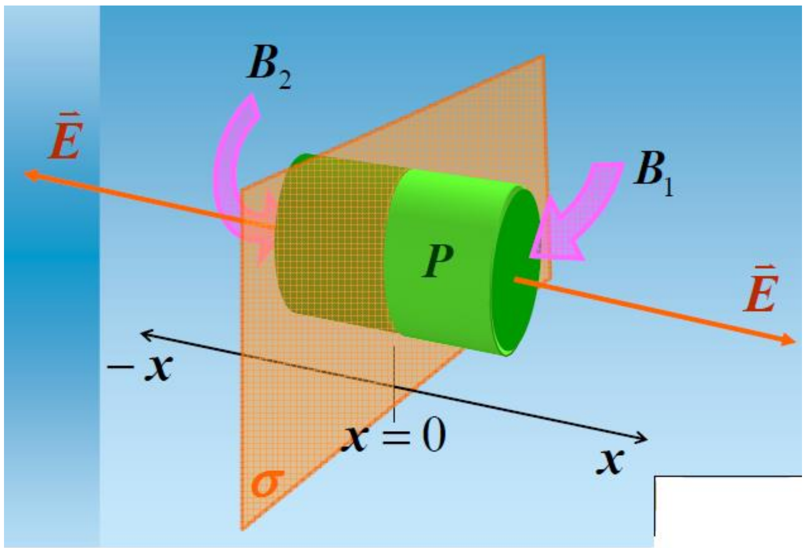
Gauss's law

$$\boxed{\Phi = \frac{Q}{\varepsilon_0}}$$



- The electric flux through any closed surface is equal to the net charge enclosed by the surface divided by permitivity of the medium.

Electric field of a plate



- application of Gauss's law to a cylinder

$$2ES = \frac{Q}{\varepsilon_0}$$

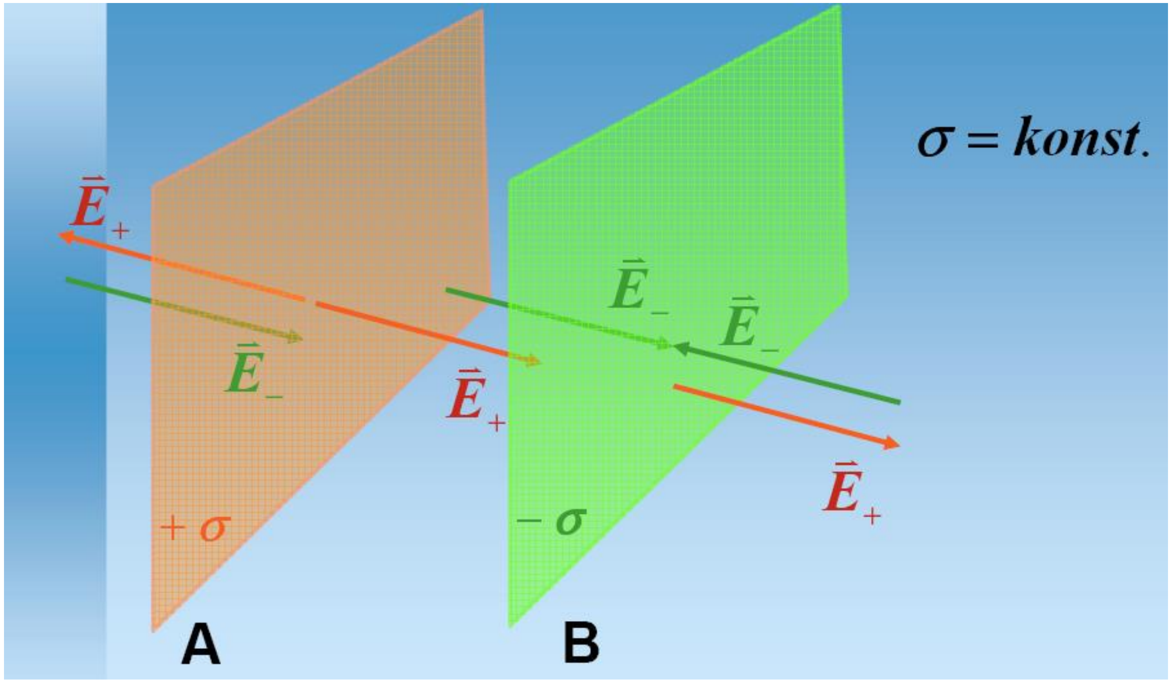
- surface density of charge

$$\boxed{\sigma = Q / S}$$

$$\boxed{E = \frac{\sigma}{2\varepsilon_0}}$$

- uniform electric field

Electric field of a plate capacitor



- net field outside of capacitor is zero
- net field inside of capacitor is:

$$E = E_+ + E_- \longrightarrow \boxed{E = \frac{\sigma}{\varepsilon_0}}$$

- flux of electric field: $\mathbf{\Phi} = \vec{\boldsymbol{E}} \cdot \vec{\boldsymbol{S}}$

- Gauss's law $\boxed{\Phi = \dfrac{Q}{\varepsilon_0}}$

- charge surface density: $\sigma = Q / S$

- uniform electric field of plate capacitor $\boxed{E = \dfrac{\sigma}{\varepsilon_0}}$

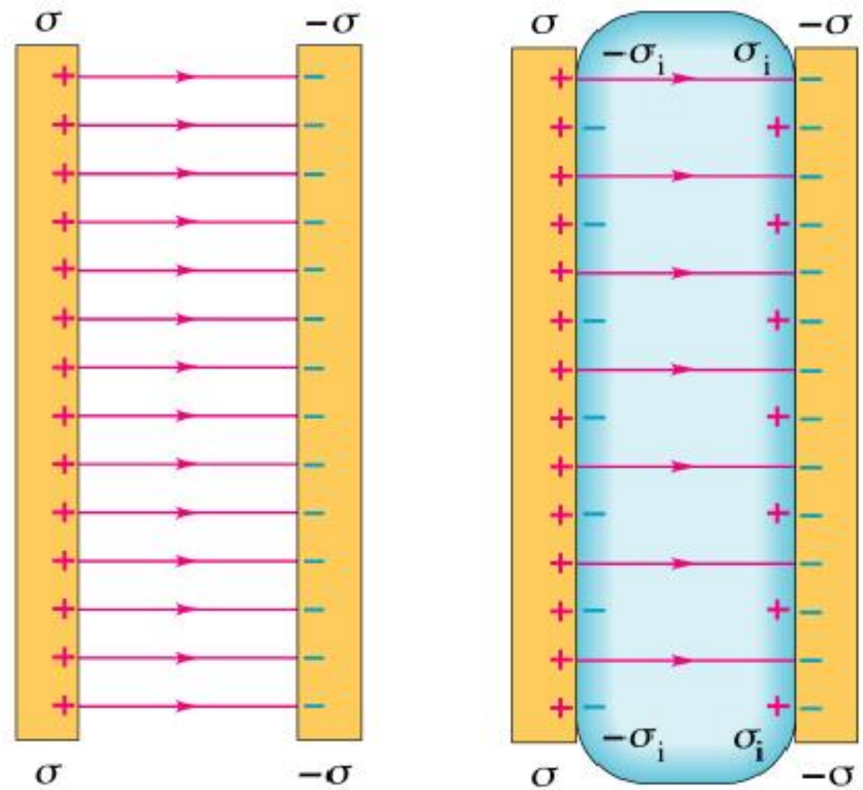
Biological tissue in external electric field?

Polarization of matter in electric field

Conductors in external eletric field

- **metals**
- **electrolytes**

- free electrons and ions
- The strength of external uniform electric field E_0

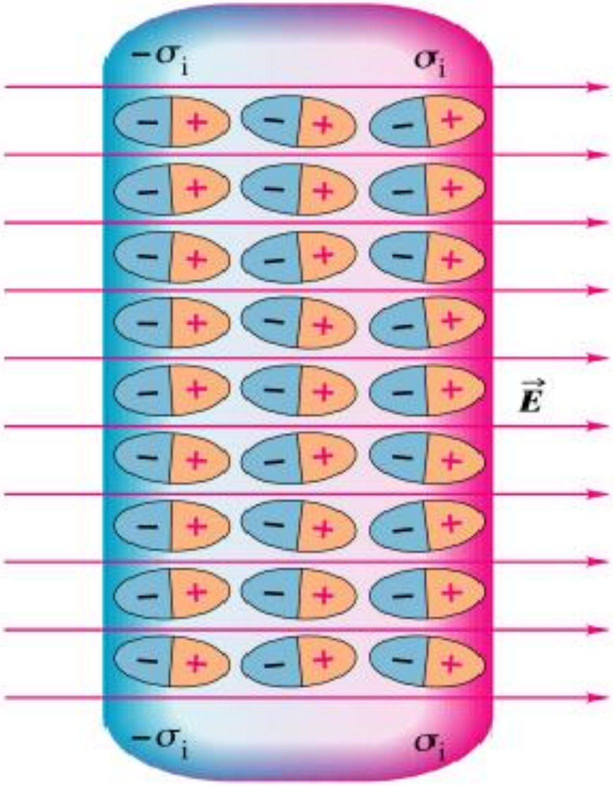


- No net electric field:

$$\boxed{E_{conductor} = -E_0}$$

Dielectrics in electric field

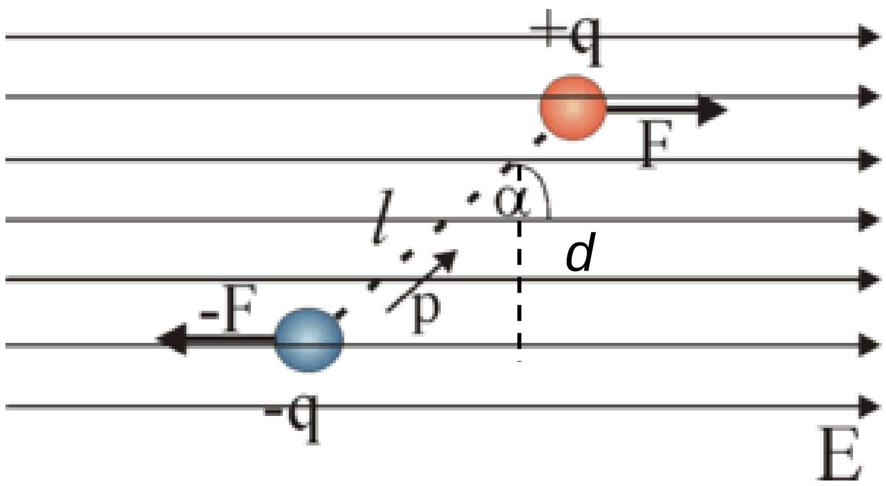
- No free charges
- Permanent and induced electric dipoles orient themselves in an external electric field
- induced electric field of dielectric: E_d



- The net electric field:

$$\boxed{E = E_0 - E_d}$$

https://www.doitpoms.ac.uk/tlplib/dielectrics/capacitors.php



- pair of forces rotates the dipole in an electric field

- torque of the pair of forces

$$\vec{\tau} = \vec{p} \, x \, \vec{E}$$

http://physics.mef.hr/Predavanja/tvariupolju/index.html

Polarization of dielectric

- Charges on dielectrics surface in external electric field

- The net electric field: $E = E_0 - E_d$

Charge density of capacitor plates

- By Gauss's law: $$\boxed{E = \frac{\sigma_0}{\varepsilon_0} - \frac{\sigma_d}{\varepsilon_0} = \frac{1}{\varepsilon_0}(\sigma_0 - \sigma_d)}$$

- Electric field of capacitor with dielectric

$$\boxed{E = E_0 / \varepsilon_r}$$

Relative permittivity defines relation between electric field of a capacitor with and without dieletric.

$$E_d = E_0 - E = \varepsilon_r E - E$$

$$\boxed{E_d = E(\varepsilon_r - 1)} \rightarrow \boxed{\chi}$$

Dielectric susceptibility

- vector of polarization – measure of polarization

$$\vec{\mathbf{P}} = \frac{\sum_{\mathbf{i}} \vec{\mathbf{p}_{\mathbf{i}}}}{\mathbf{V}}$$

- charge surface density $\boxed{P = \sigma_d}$

$$\boxed{P = \varepsilon_0 \mathrm{E}_d = \varepsilon_0 \chi \mathrm{E}}$$

Relative permittivity

$$\varepsilon_r = \frac{E_0}{E} \geq 1$$

- high relative permittivity – electric dipoles are oriented in the electric field better

matter	ε_r
organic solvents	2 - 5
fat tissue	12
glass	6 - 10
cow's milk	66
water	81
white tissue - brain	90
gray tissue - brain	85

- Measure of the polarization efficiency – relaxation time τ

Matter in an electric field

conductors

$$E = 0$$

dielectrics

$$E = E_0 - E_d$$

- polarization

$$\boxed{\sigma_d = \varepsilon_0 E_d}$$

- relative permittivity: $\boxed{\varepsilon_r = E_0 / E}$

- Dielectric susceptibility: $\boxed{\chi = E_d / E}$

Summary II

Mechanisms of polarization

In a homogeneous substance

1. **DIPOLE**
2. **ELECTRONIC**
3. **IONIC**

inhomogeneous substances in additional mechanisms are:

4. **PHASE**
5. **MEMBRANE**
6. **ELECTROLYTIC**

Dipole polarization

- It occurs in compounds with polar molecules.

http://physics.mef.hr/Predavanja/tvariupolju/index.html

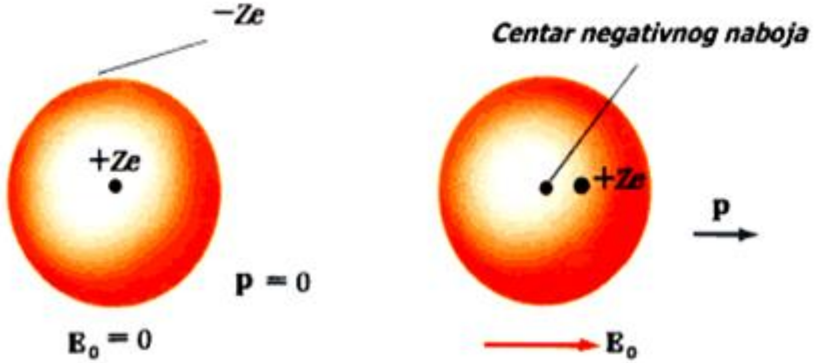
- Polar molecules are directed toward the field.

- Directing is not perfect due to the chaotic thermal motion.
  - Relaxation time: $\boxed{\tau = 10^{-10}\,\mathrm{s}}$

http://www.doitpoms.ac.uk/tlplib/dielectrics/dielectric_constant.php

Electronic polarization

- It occurs in compounds without polar molecules.



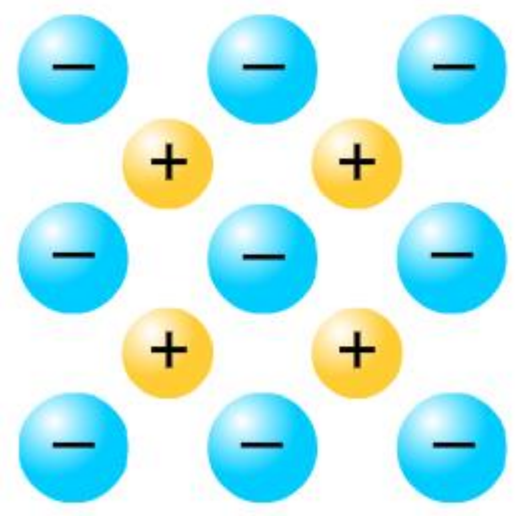
- Electronic cloud is deformed when they are inserted in the external field.
- induced dipole moment: $\boxed{p \propto E_0}$
- Relative permittivity does not depend on temperature
- Relaxation time: $\boxed{\tau = 10^{-15}\,\mathrm{s}}$

http://www.doitpoms.ac.uk/tlplib/dielectrics/dielectric_constant.php

Ionic polarization

- It occurs in ionic crystals.

- The external electric field causes tha spacing of ions in crystals with ionic bonds. Shifts are within the dimensions of molecules.



- Relaxation time:

$$\boxed{\tau = 10^{-13}\mathrm{s}}$$

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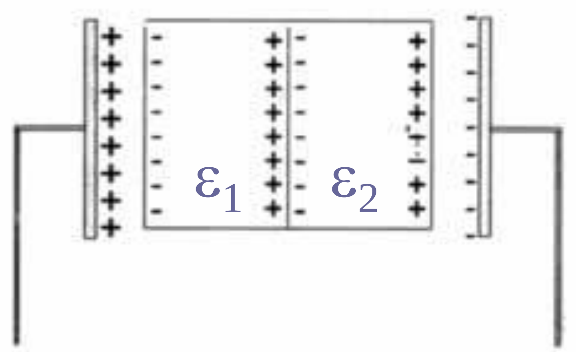
Phase polarization



- The appearance of the surfae charge on the tissue regions with the different dielectric properties.

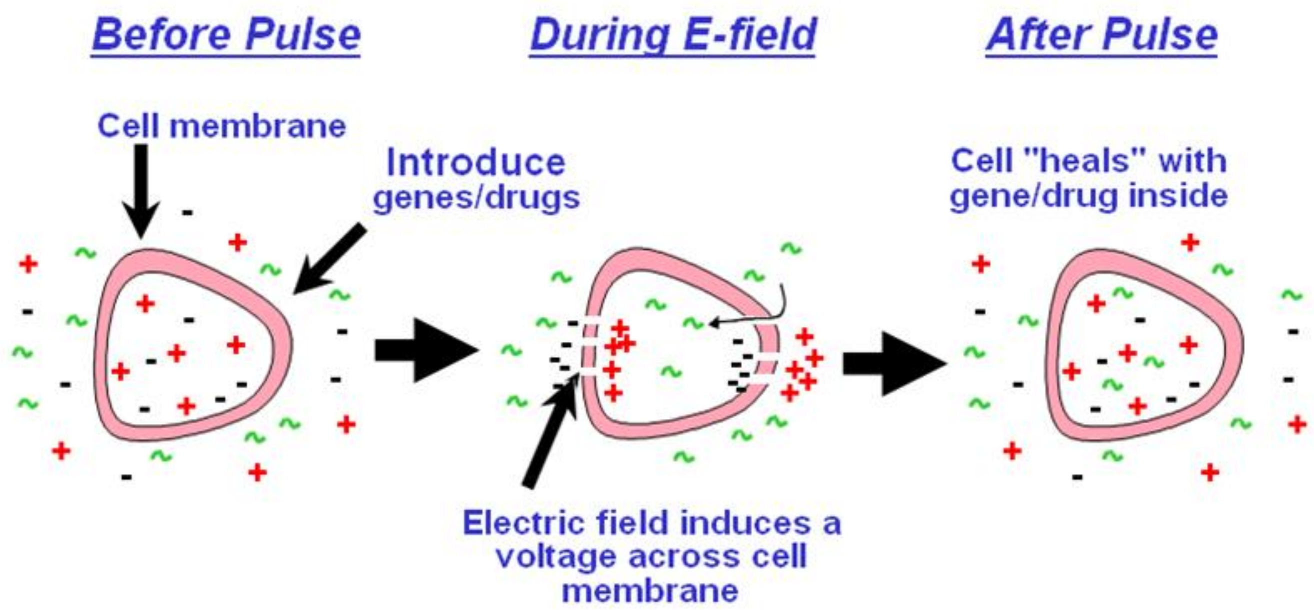
$$\boxed{\tau = 10^{-3} s}$$

Membrane polarization

- When the tissue in the external electric field, the membrane polarization can be either in the direction of the field or in opposite direction.

- **It is very important for living tissue.**

- Relaxation time: $\boxed{\tau = 10^{-2} s}$



http://www.intechopen.com/books/advances-in-micro-nano-electromechanical-systems-and-fabrication-technologies/electroporation-based-drug-delivery-and-its-applications

http://www.medgadget.com/2007/08/treating_cancer_with_electric_fields.html

Electrolyte polarization

- The whole body acts as a large induced dipole.

- Relaxation time:

$$\boxed{\tau > 1\ s}$$

Polarization of matter in alternating electric field

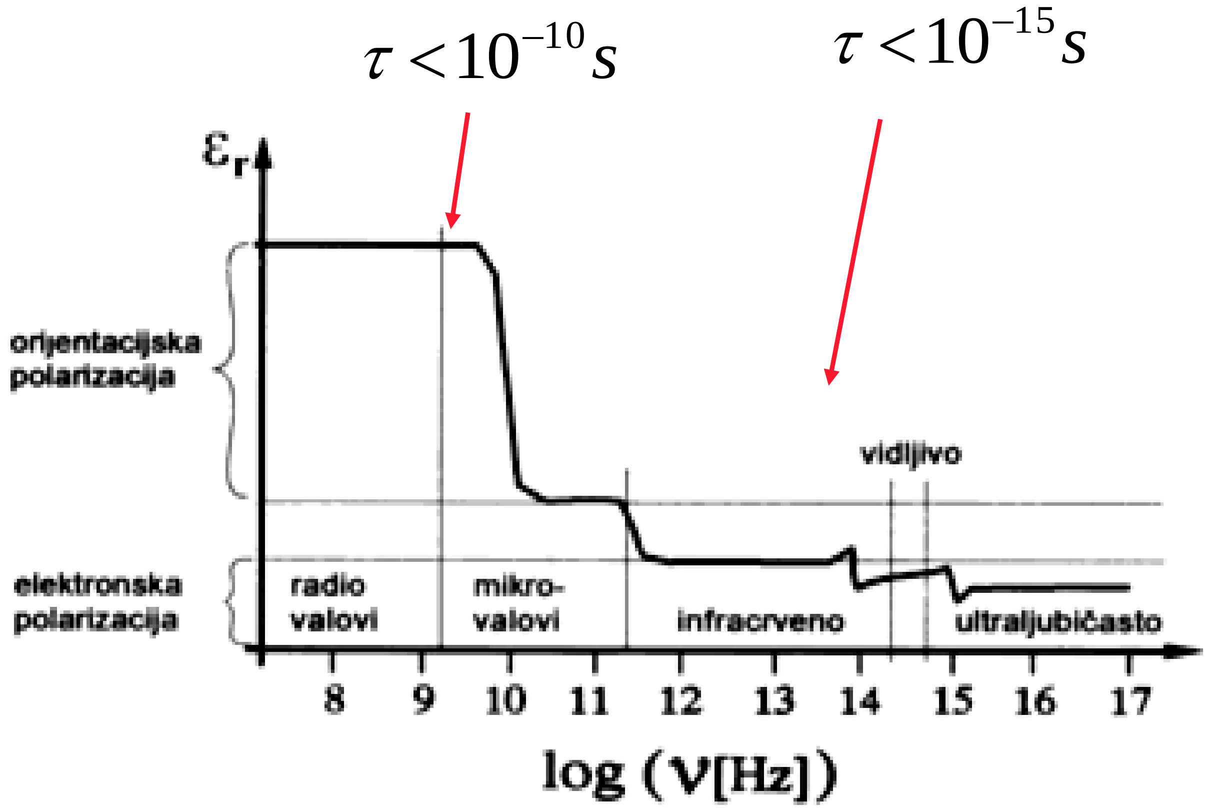
- Time independent electric field of capacitor $\longrightarrow$ $\varepsilon_r = konst.$
- alternating electric field of capacitor $\longrightarrow$ $\varepsilon_r(\mathrm{f}\ )$

$$\boxed{E = E_0 \sin \omega t}$$

$$\boxed{\tau \leq \frac{T}{2}}$$

$$\omega = 2\pi\ \mathrm{f} \qquad \mathrm{T} = \frac{1}{\mathrm{f}}$$

Polarization of matter in an alternating electric field



- relative permittivity as a function of frequency of external electric field
- *Dielectric function*

Relaxation time τ (s)	mechanism of polarization
10^{-15}	electronic
10^{-10}	dipole
10^{-3}	phase
10^{-2}	membrane
1	electrolyte

- mechanism of polarization is defined by time relaxation

$$\boxed{\tau \leq \frac{T}{2}}$$

Summary III