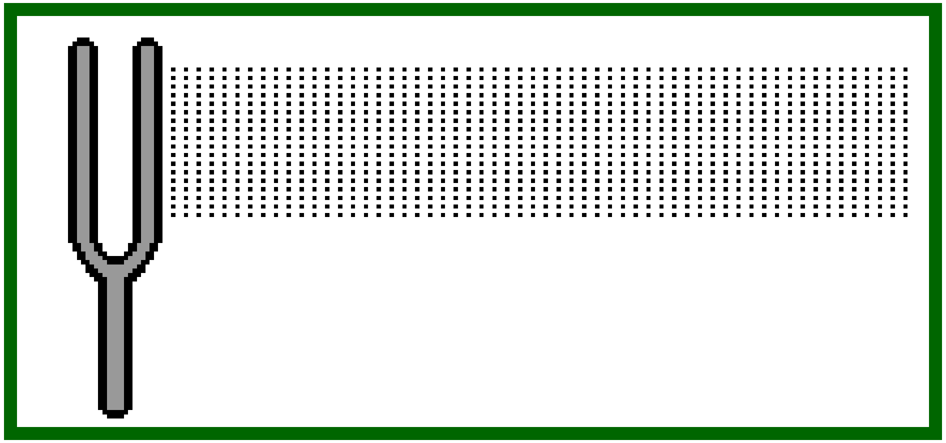
Sveučilište u Zagrebu
MEDICINSKI FAKULTET

VIBRATION AND SOUND



Sanja Dolanski Babić

Vibration and sound wave

- **How do we hear?**
- Because of interaction of sound waves and our ears

- infrasound, sound (20 Hz – 20 kHz), ultrasound

Sound wave – transfer of mechanical energy by vibration of elastic medium particles

Simple harmonic motion

- Elastic force $F_{el} = -kx$
- II Newton's law $F = ma$

$$m\frac{d^2x}{dt^2} = -kx$$

$$m\frac{d^2x}{dt^2} + kx = 0$$

$$T = 2\pi\sqrt{\frac{m}{k}}$$

$$\omega = \frac{2\pi}{T}$$

$$x(t) = A\sin\omega_0 t$$

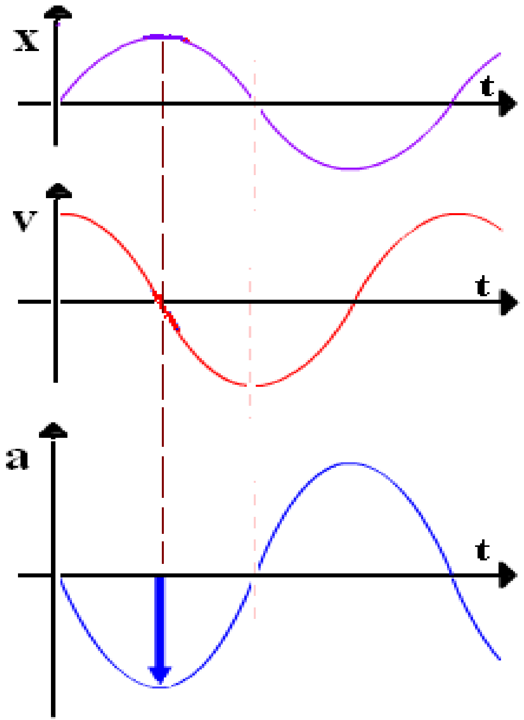
Starting values:x=0 za t=0

Own angular frequency

$$\omega_0 = \sqrt{\frac{k}{m}}$$

http://www.animations.physics.unsw.edu.au/jw/SHM.htm

Simple harmonic motion



$$x = A$$

$$x(t) = A\sin\omega_0 t$$

$$\mathrm{v} = 0$$

$$\mathrm{v} = \frac{dx}{dt} = \omega_0 A\cos\omega_0 t$$

$$a = -\omega_0^2 A$$

$$a = \frac{d\mathrm{v}}{dt} = -\omega_0^2 A\sin\omega_0 t$$

$$E = \frac{m\mathrm{v}^2}{2} + \frac{kx^2}{2}$$

⇒

$$E = \frac{m\omega_0^2 A^2}{2}$$

http://www.animations.physics.unsw.edu.au/jw/SHM.htm

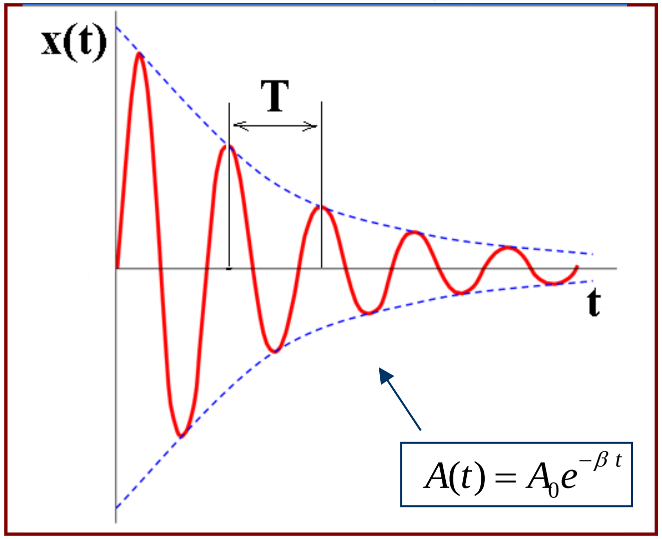
Damped oscillations

- Friction force: $$F = -r \cdot \mathrm{v}$$

  - Energy is decreasing

  - Amplitude is decreasing

$$m\frac{d^2x}{dt^2} = -kx - r\mathrm{v}$$



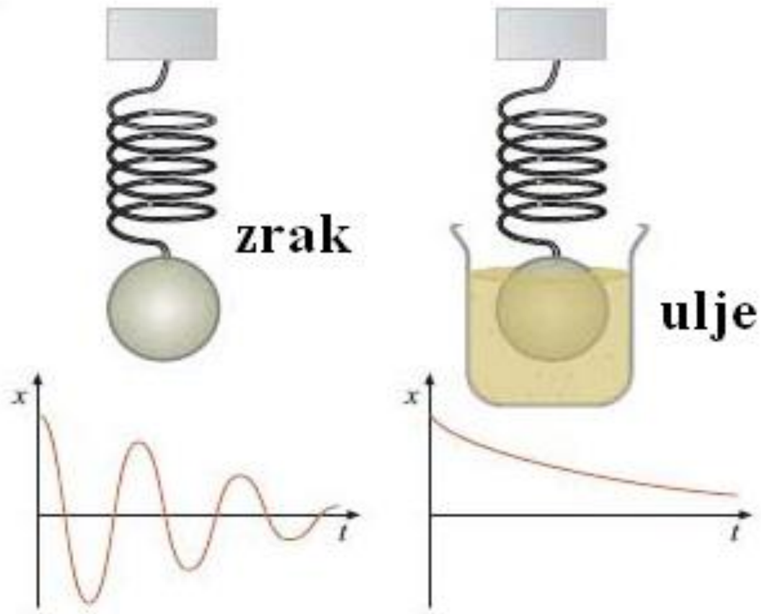
$$x(t) = A_0 e^{-\beta t} \sin \omega t$$

$A(t)$

$$\beta = \frac{r}{2m}$$ Damping coefficient

- Smaller than angular frequency of simple pendulum motion

$$\omega = \sqrt{\omega_0^2 - \beta^2}$$



$$\beta > \omega_0$$

Forced harmonic oscillations

- The object could be forced to harmonic oscillation by action of external harmonic force

$$F = F_0 \sin \omega t$$

- The object will oscillate with the frequency of external force.

http://www.animations.physics.unsw.edu.au/jw/oscillations.htm#Forced

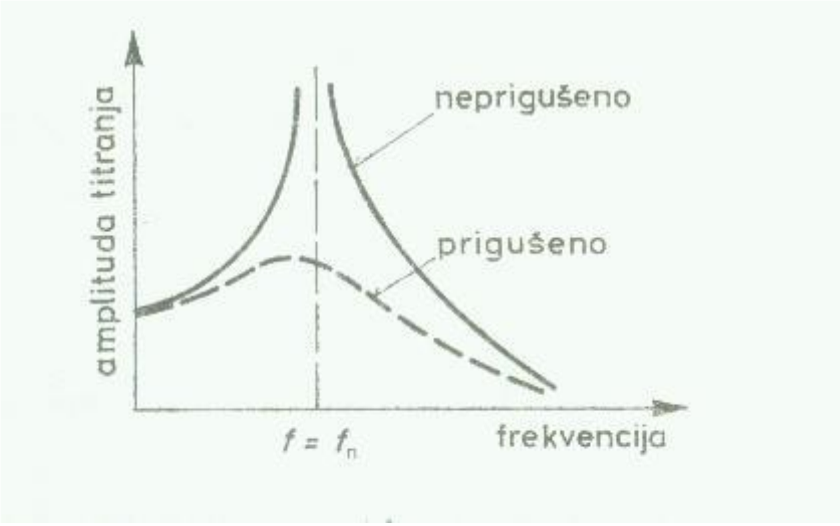
$$m\frac{d^2x}{dt^2} = -kx - r\mathrm{v} + F_0 \sin \omega t$$

- The general solution is very complex.

$$x(t) = A(\omega) \sin \omega t$$

- Amplitude depends of frequency ratio ω / ω_0 and of damping coefficient β

- Maximum of amplitude –resonant frequency



- Resonant frequency $$\omega_r = \sqrt{\omega_0^2 - 2\beta^2}$$

- For β=0: $$\omega_r = \omega_0$$

http://www.youtube.com/watch?v=dU0OqVDI7kc&feature=related

Summary

oscillation – periodical motion

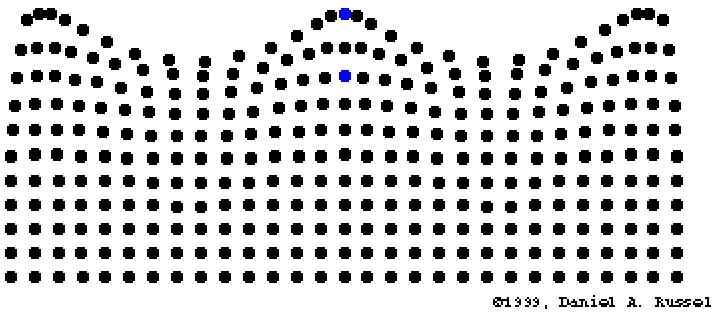
harmonic oscillations	frequency ω	amplitude A
free	ω_0	$A = A_0$
damped	$\omega = \sqrt{\omega_0^2 - \beta^2}$	$A(t) = A_0 e^{-\beta t}$
forced - resonance	$\omega_r = \sqrt{\omega_0^2 - 2\beta^2}$	$A(\omega)$

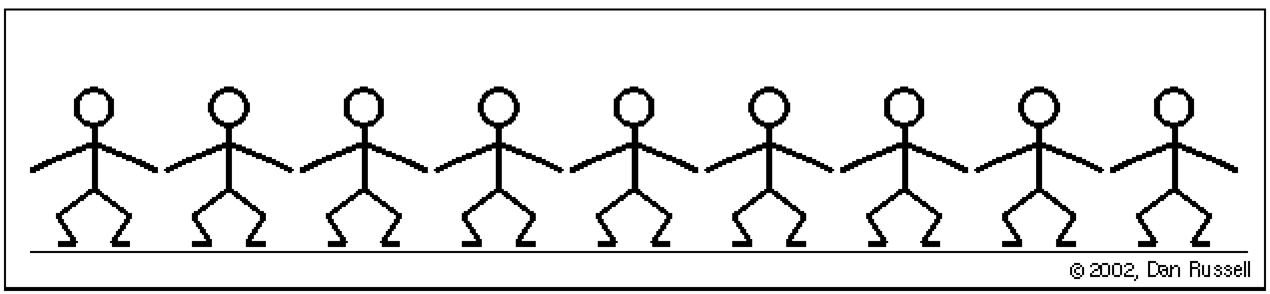
$$E = \frac{m\omega_0^2 A^2}{2}$$

Sound waves – transfer of mechanical energy

Mechanical waves

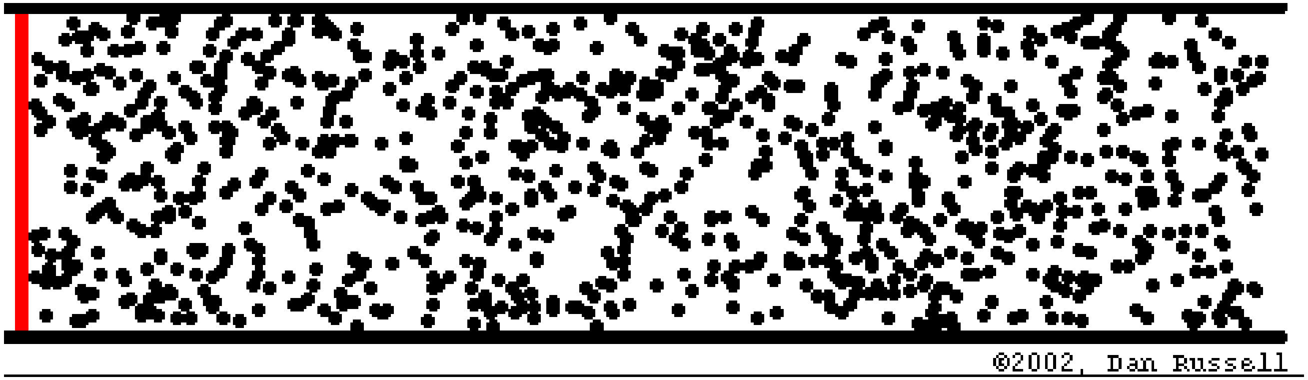
puls





preuzeto: http://paws.kettering.edu/~drussell/Demos/waves-intro/waves-intro.html

Nature of sound wave



- **source** – body which vibrates in elastic medium
- Energy is spreading with the speed

$$\mathrm{v} = \frac{\lambda}{T} = \lambda \cdot f$$

- **Acoustic pressure**

Intensity of sound wave

$$I = \frac{E}{St}$$

$$E = \frac{m\omega_0^2 A^2}{2}$$

$$m = \rho \cdot V$$

$$I = \frac{\omega_0^2 A^2 \rho \mathrm{v}}{2}$$

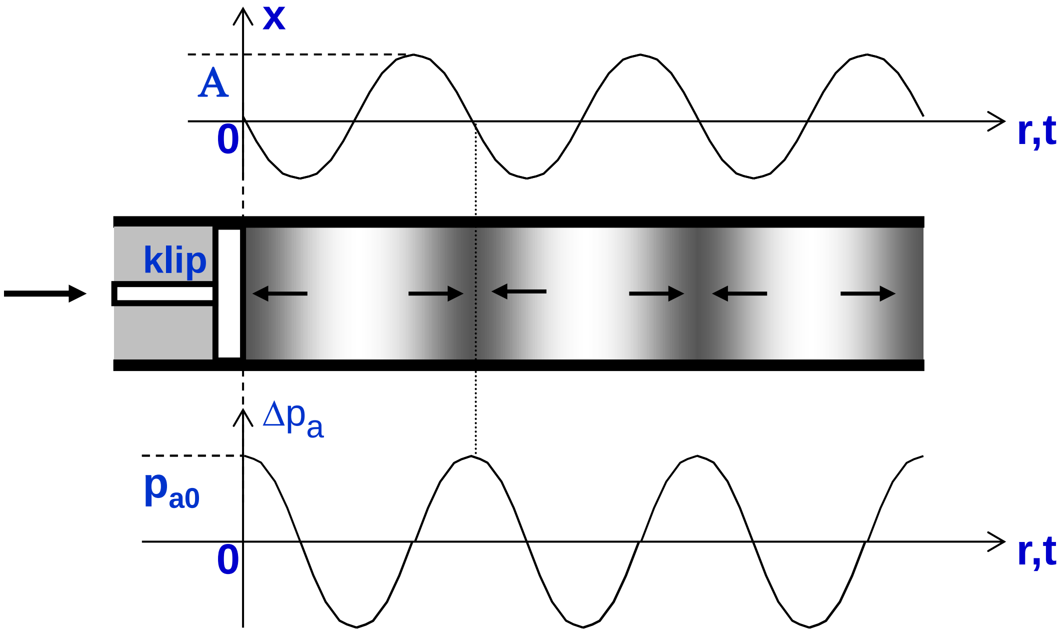
- intensity depends of source properties *(native frequency and amplitude)* and elastic medium (acoustic impedance*,* $Z = \rho\ \mathrm{v}$*)*

http://www.youtube.com/watch?v=DanOeC2EpeA

Wave equation

- periodic in time and space

$$x(r,t) = A\sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda}\right)$$



$$10^{-5}\,Pa \leq p_{a0} \leq 10\,Pa$$

$$\Delta p_a(r,t) = p_{a0}\sin 2\pi\left(\frac{t}{T} - \frac{r}{\lambda} - \frac{\pi}{2}\right)$$

$$I = \frac{p_{a0}^2}{2Z}$$

Intensity level

- the range of intensities which the human ears can detect is very large (10^{-12} W/m^2 – 1 W/m^2) and absolute intensity is not needed in considerations
- intensity level:

$$\beta = 10 \log \frac{I}{I_0}$$

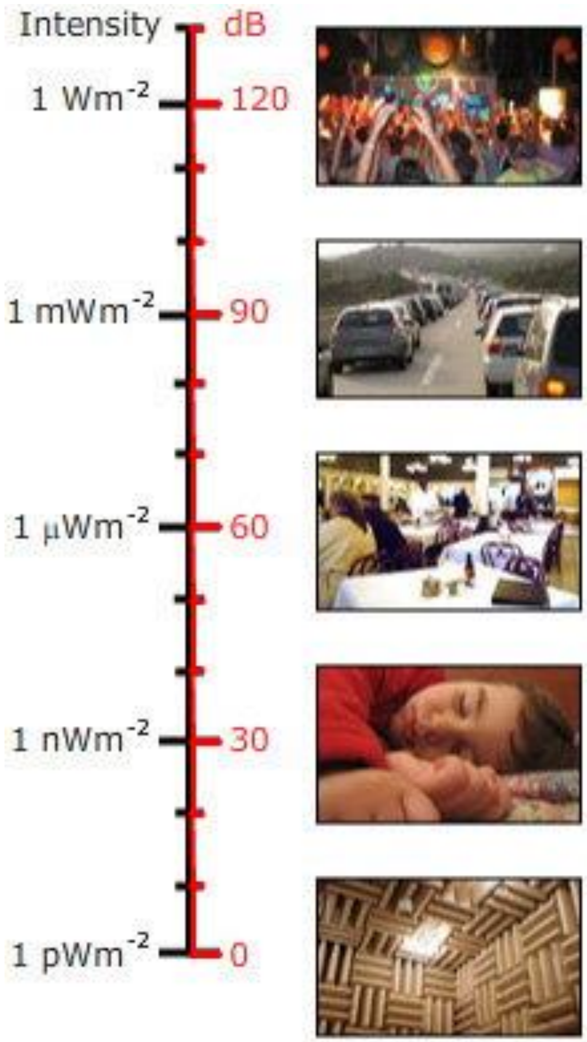
- referent intensity: 10^{-12} W/m^2

  - treshold of hearing on f=1000 Hz

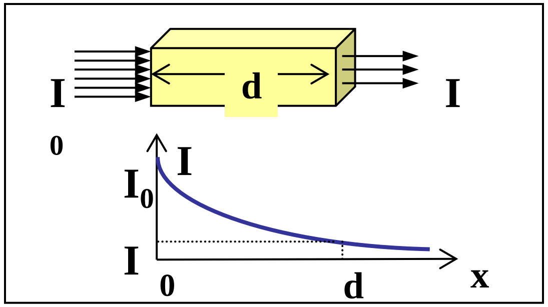
- unit: dB

Table intensity for tone on 1 kHz $\beta = 10 \log \frac{I}{I_0}$

sound	I/Wm^{-2}	β/dB
treshold of hearing	10^{-12}	0
rustling leaves	10^{-11}	10
whisper	10^{-10}	20
normal conversation	10^{-8}	40
busy street traffic	10^{-6}	60
vacuum cleaner	10^{-4}	80
front rows of rock concert	10^{-2}	100
treshold of pain	1	120



Absorption of sound wave



- Interaction sound waves and matter
- Damped oscillation
- Amplitude is decreasing: $$A = A_0 e^{-\alpha x}$$

α *– linear coefficient, depends of type of matter and frequency*

- Intensity: $I \propto A^2$

$$I = I_0 e^{-2\alpha x}$$

- Half thickness: $x_{1/2} \rightarrow I_0/2$

$$x_{1/2} = \frac{\ln 2}{2\alpha}$$

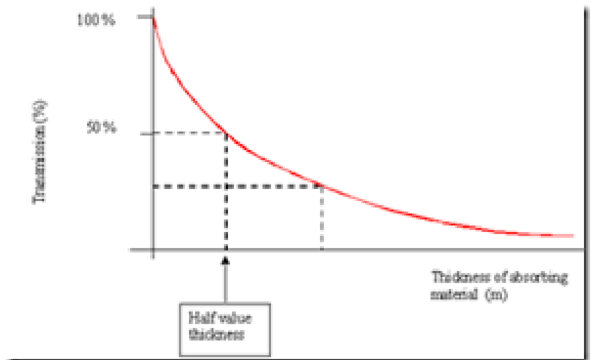
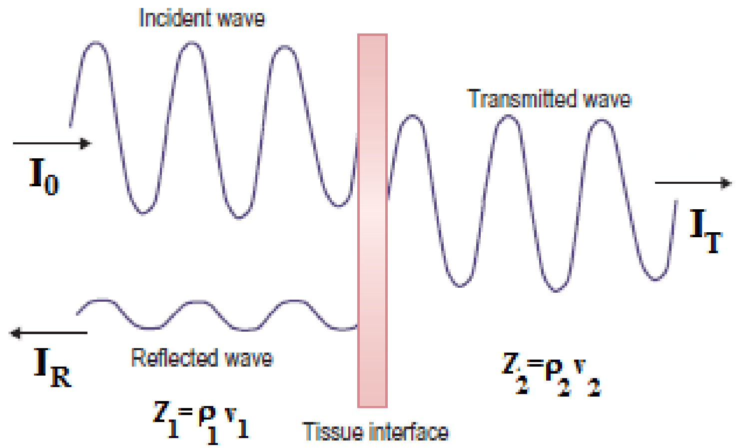


Table 1.5 Tissue thickness (in cm) to half intensity of ultrasound beam (–3 dB) for common clinical frequencies and materials

	1 MHz	3 MHz	5 MHz	10 MHz	20 MHz
Blood	17	8.5	3	2	1
Fat	5	2.5	1	0.5	0.25
Liver	3	1.5	0.5	–	–
Muscle	1.5	0.75	0.3	0.15	–
Bone	0.2	0.1	0.04	–	–
Polythene	0.6	0.3	0.12	0.6	0.03
Water	1360	340	54	14	3.4
Soft tissue (average)	4.3	2.1	0.86	0.43	0.21



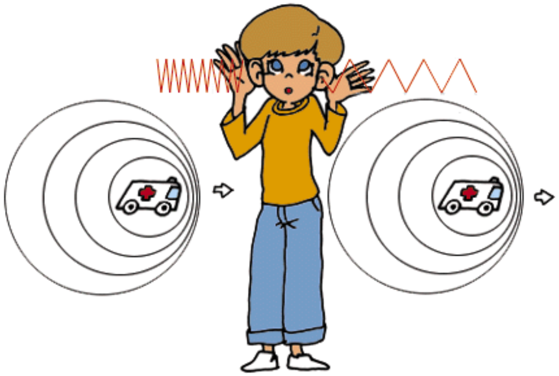
$\alpha_1 = \alpha_2 = 0^o$

$$\frac{I_R}{I_0} = \frac{(Z_2 - Z_1)^2}{(Z_2 + Z_1)^2} \qquad \frac{I_T}{I_0} = \frac{4Z_1Z_2}{(Z_2 + Z_1)^2}$$

- za $Z_1 \cong Z_2$ max. transmission!
- za $Z_1 >> Z_2$ or $Z_2 >> Z_1$ max. reflection!

- Z(air) = 430 kg/m^{2}s, Z(water) = 1 480 000 kg/m^{2}s

Doppler effect



- Consequence of relative movement of source or detector of sound is virtual change of frequency of the detected sound

- approaching to the source ⇨ higher frequency

$$f_p = f_i \frac{v_z + v_p}{v_z}$$

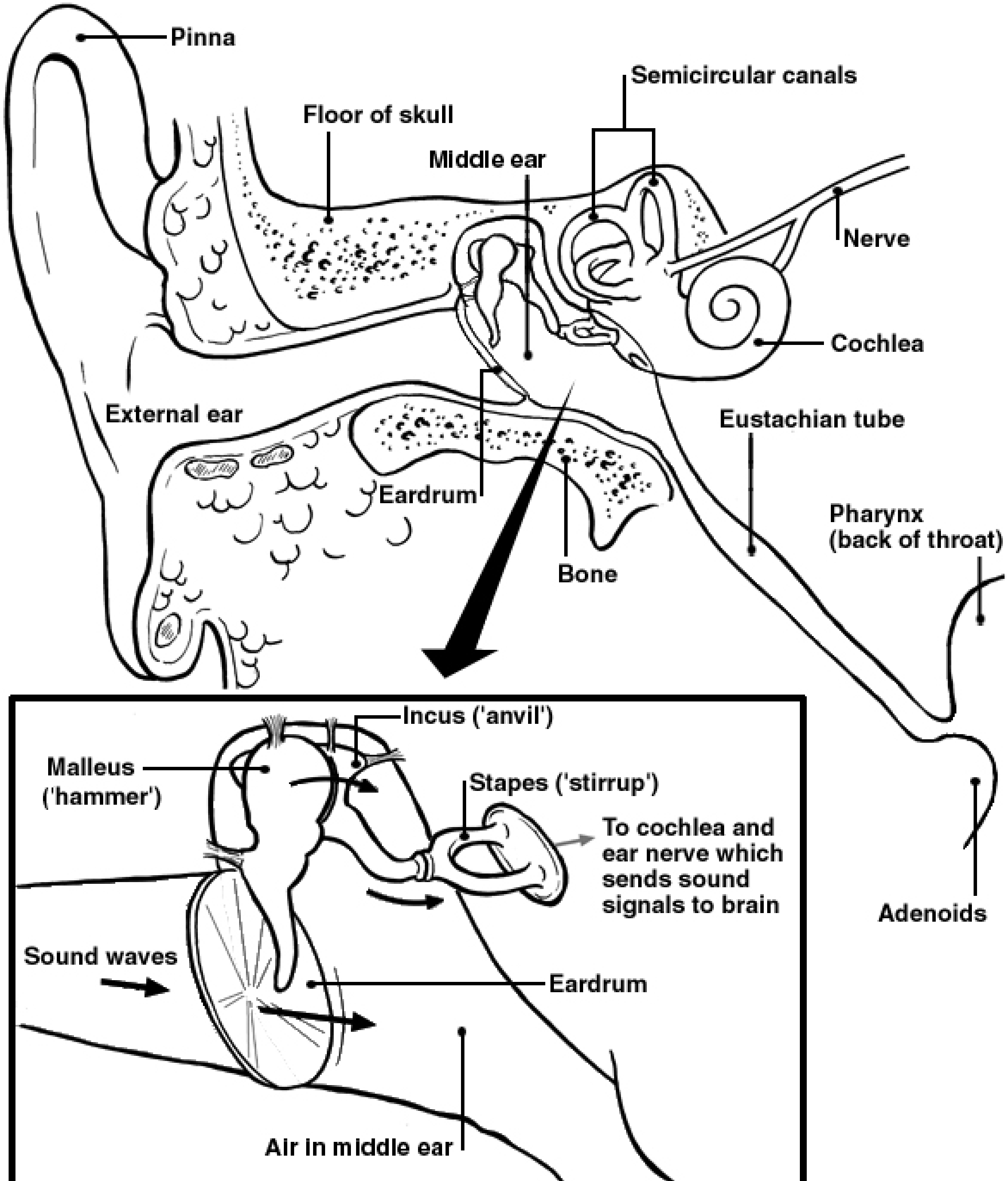
- to go away from the source ⇨ lower frequency

$$f_p = f_i \frac{v_z - v_p}{v_z}$$

- simultaneously approaching

$$\Delta f = f_0 \frac{v_p - v_i}{v_z}$$

http://www.animations.physics.unsw.edu.au/jw/doppler.htm

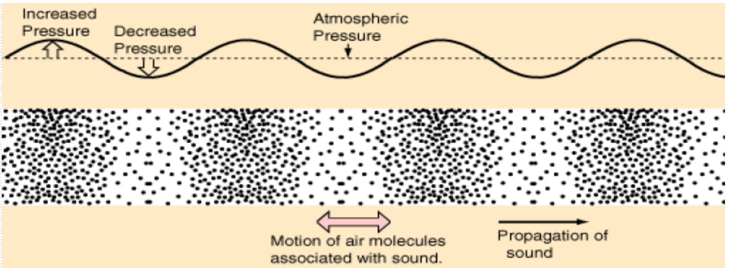


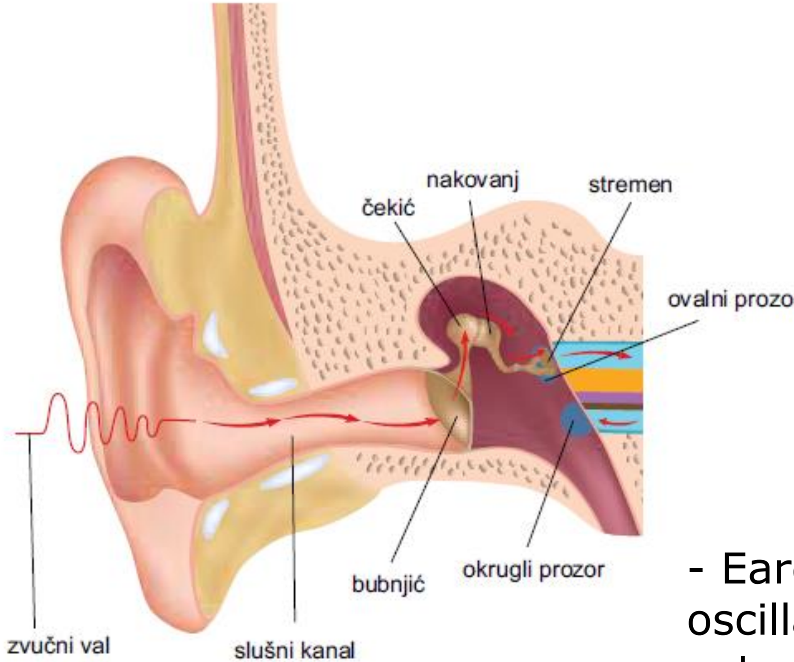
External ear – sound waves enter

Middle ear – amplification of sound waves

Inner ear – frequencies parsed, transduced to electrical impulse and sand to brain

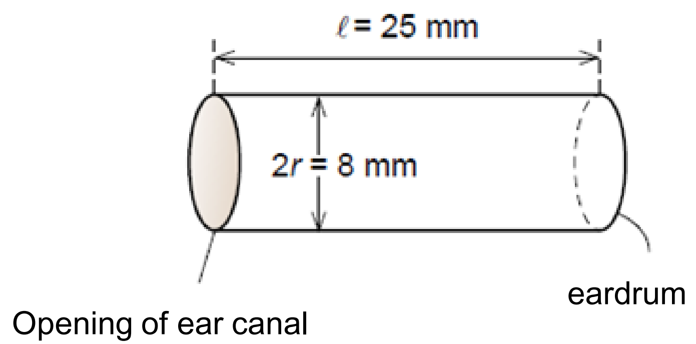
External ear





- the change of acoustic pressure

ear canal – tube closed on one side; 2.5 cm long – 0.8 diameter; the resonance frequency is 3.3. kHz (wave length – 10 cm)



- Eardrum transmits oscillations from the external to the middle ear
- Displacement of eardrum at hearing threshold - **0.01 nm** amplitude of pain – **11 mm**
Compare! Diameter of cell ~ 10 mm
in disco club (140 dB) – **displacement of eardrum is 100 mm**

Acoustic parameters - the physiological sensations

- **intensity level - loudness level**
- **frequency - sound pitch**
- **frequency spectrum - sound quality (tone timber)**

Loudness level

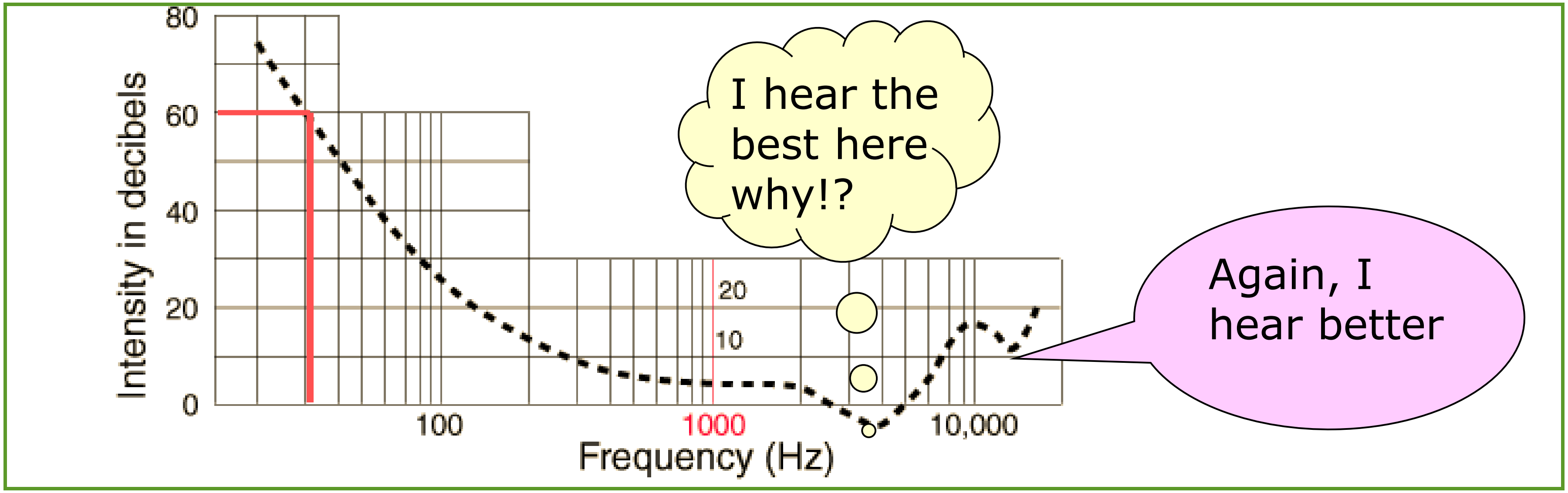
- Weber-Fechnerov law(for 1000 Hz i $I_0 = 10^{-12} Wm^{-2}$):

$$S = 10 \log \frac{I}{I_0}$$

- Loudness unit phon - for tone 1 kHz the change of intensity level for 10 dB results in loudness level change of 10 phons

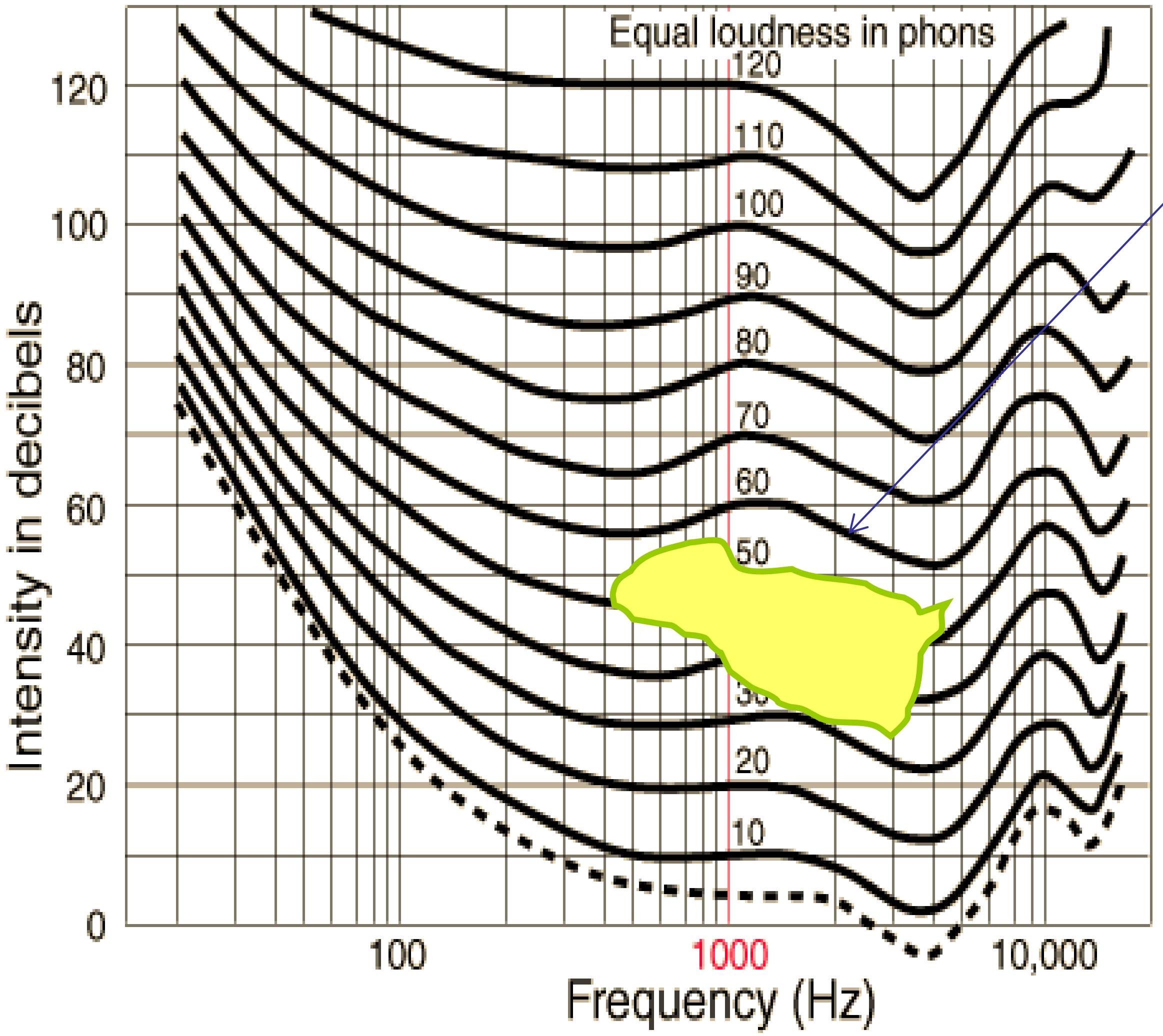
http://www.animations.physics.unsw.edu.au/waves-sound/quantifying/

Hearing threshold



- hearing threshold for 1000 Hz -theory 0 dB, measured 4 dB
- discrimination on low frequencies – tone of 30 Hz needs the intensity of 60 dB
- maximal sensitivity of the ear on 3500 to 4000 Hz because the resonance in auditory canal
- can be modeled as closed tube resonances of the auditory canal

Isophonic curves equal loudnes curves



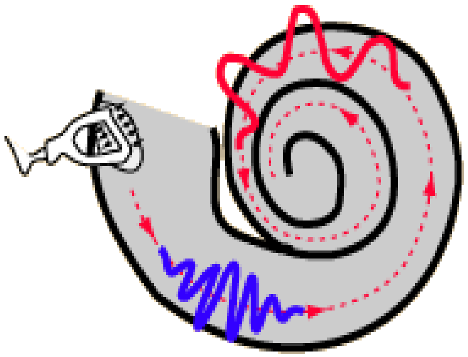
- the interval of speech:
- frequency: 0,25 - 3 kHz
- loudness: 30 - 60 phon
- best hearing: 3,5-4 kHz

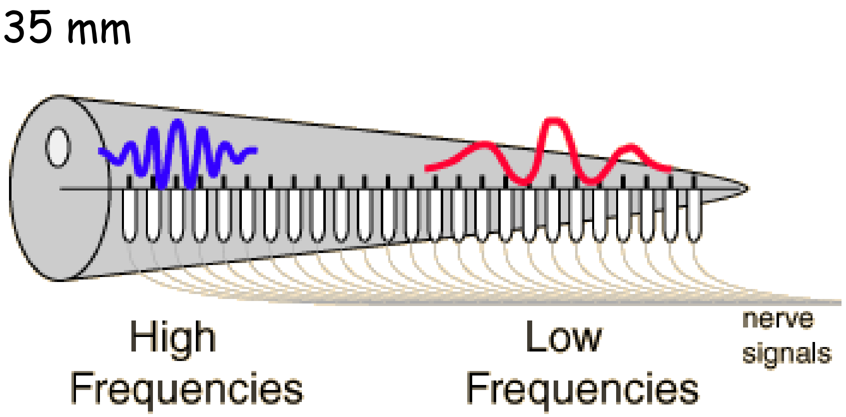
- **we do not hear sounds of:**
- - blood flow in head,
- - displacement of the jaw joint
- - Brownian motion of air molecules

Pitch of a sound

- **Tone pitch ~ $\log_2 f$**

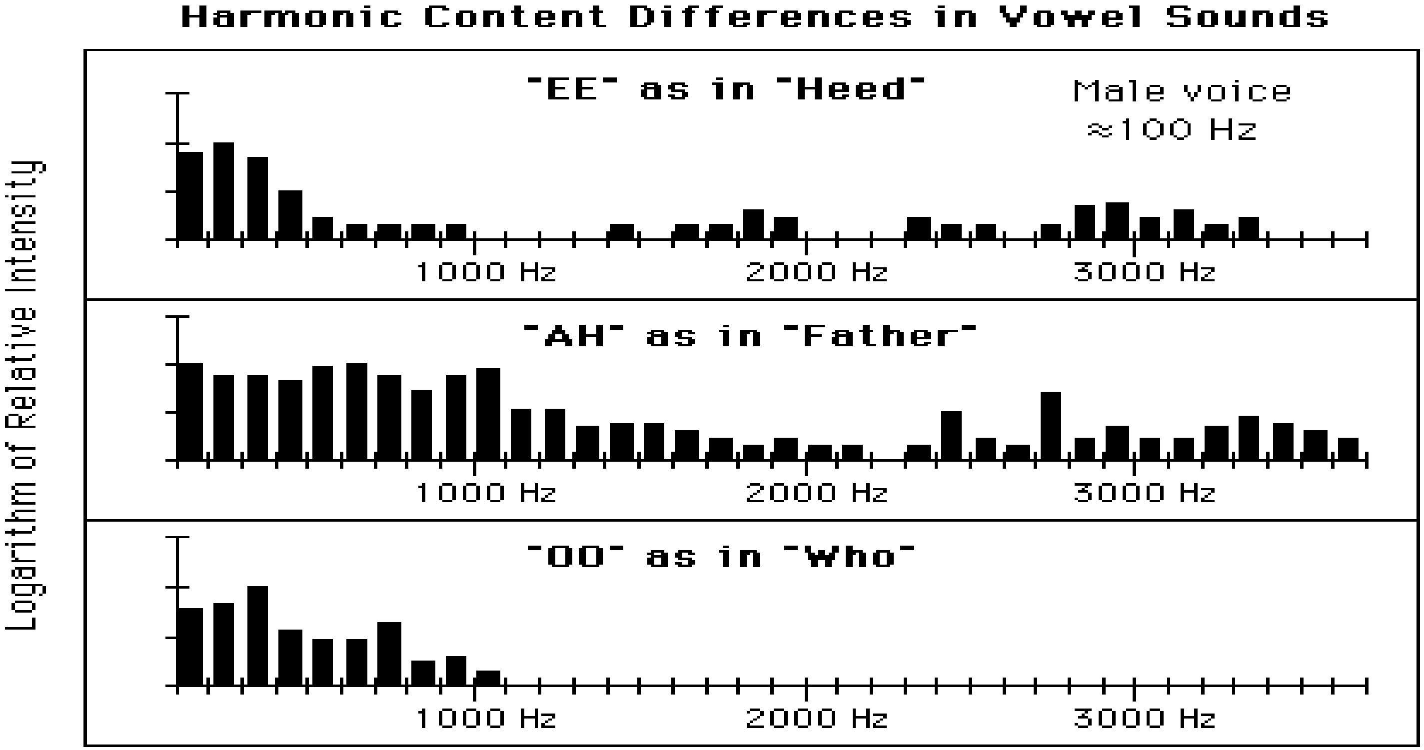
- sensation of pitch explained with theory of place and resonance of sensation cells - high resolution of our ear for pitch tone
- satisfactory explanation of sensation of relative pitch tone, but not the explanation of absolute pitch
- discrimination of pitches is high - between 0.08 i 10 kHz dependent





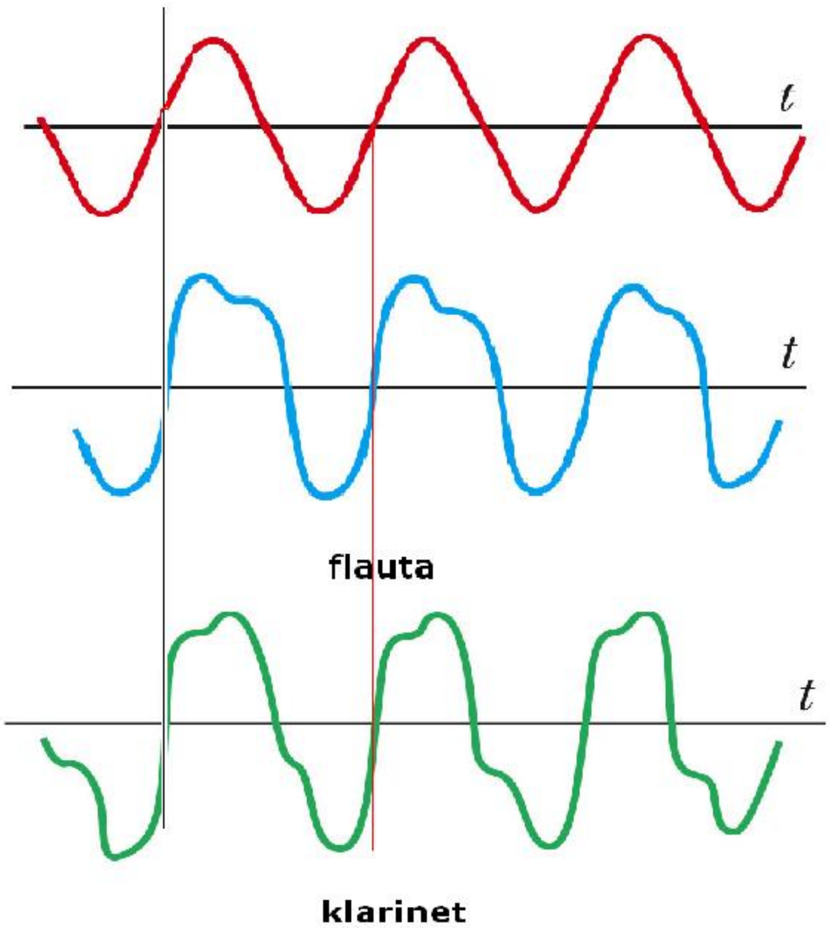
Timbre – sound quality

- our ear distinguishes sounds of same pitch and loudness
- we distinguish among different instruments



- **Sound waves: tones and noise**

- tones: simple – harmonic

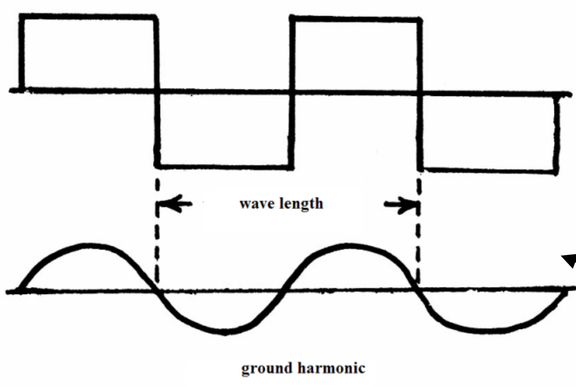


equal frequencies

complex - nonharmonic

http://www.animations.physics.unsw.edu.au/jw/SHM.htm

Nonharmonic tones



- nonharmonic function
- harmonic function

- Fourier theorem – nonharmonic function can be represented by sum of harmonic functions

$$x(t) \approx \sum_{i=1}^{n} A_i \sin \omega_i t$$

$$\omega_i = i \cdot \omega_1$$

- harmonic contents: dispersion of amplitudes upon frequencies

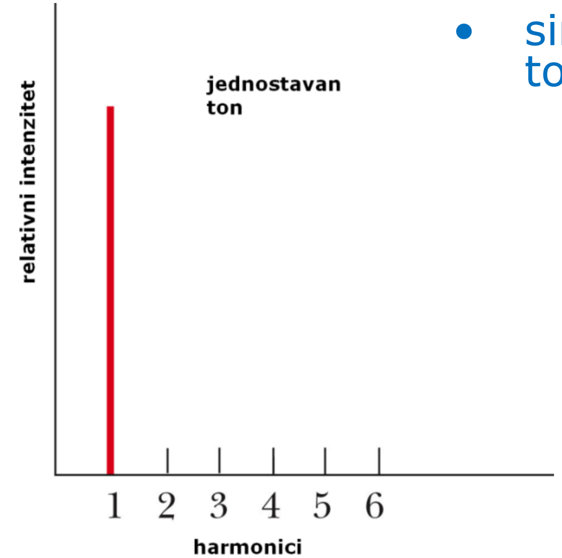
- frequency of higher harmonic
- frequency of ground harmonic

- Fourier theorem

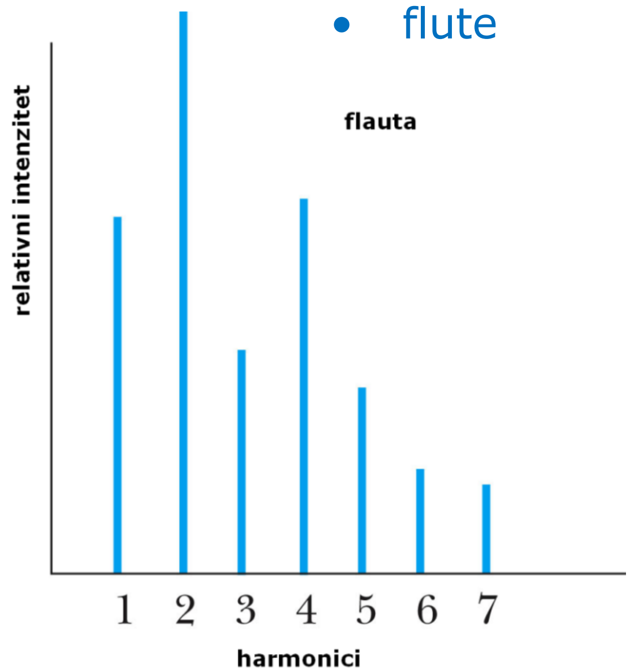
$$x(t) \approx \sum_{i=1}^{n} A_i \sin \omega_i t \qquad \omega_i = i \cdot \omega_1$$

- distribution amplitudes per harmonics

- simple tone



- flute



- clarinet