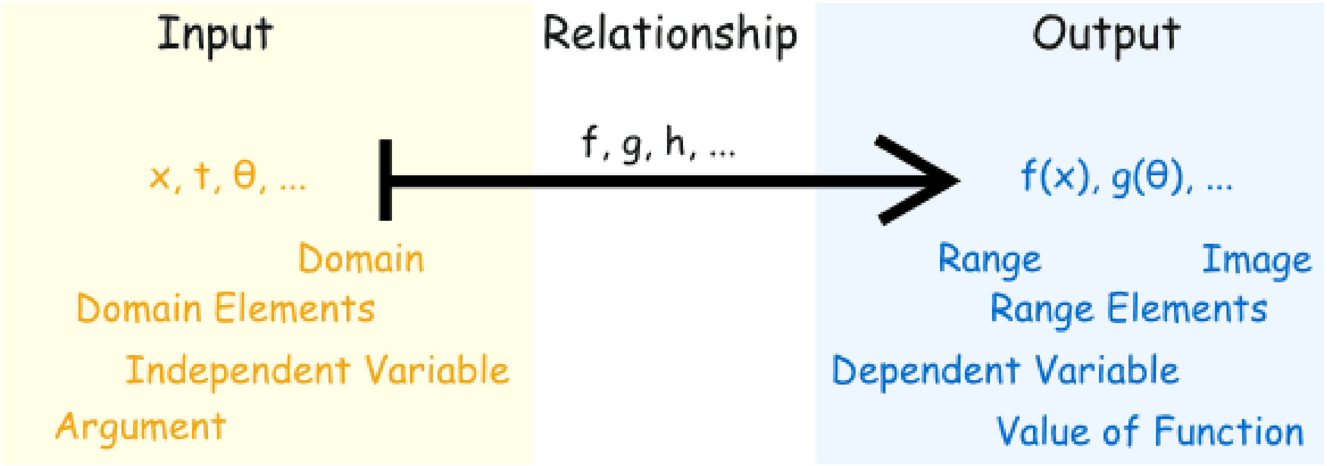
Math functions



April,2018.

S.Dolanski Babić

- Variables – quantitive description of chosen system properties
- Physical law – describes connection between variables of one or several observed systems
- Function – mathematical procedure of assignation of variables from one set to the variables from the other

Function

- general form: $y = f(x,t)$
- independent variables: x, t
- dependent variable: y
- function, f – describes how variable y depends on variables x and t

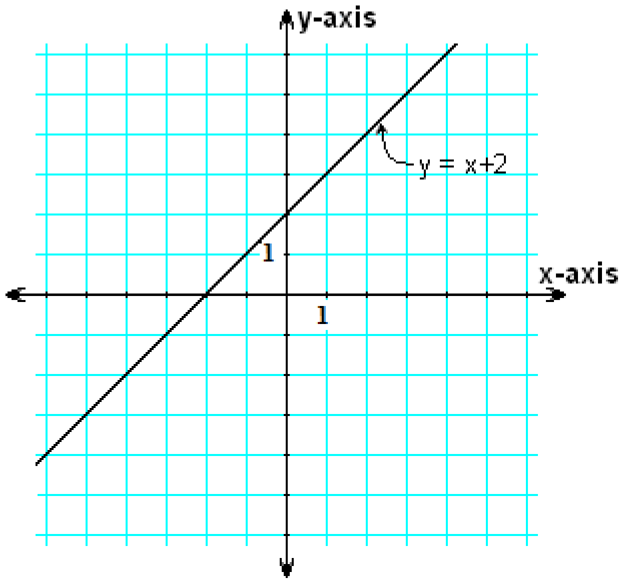
Function

- Modes function are:

1. graphical in coordinate frame
2. tabulation – two interconnected sequences of numbers
3. analytical review – mathematical equation

Linear function

- mathematical representation is: $y = ax + b$
- graphical representation is straight line



- linear function may be:
  - increasing for a > 0
  - decreasing for a < 0

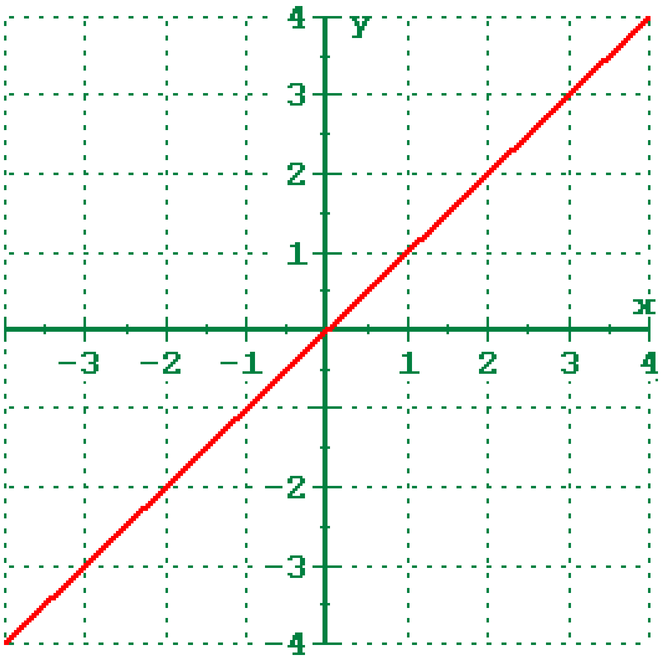
- zero point has coordinates:

$$x = -b / a$$

- a – slope
- b – y-intercept

Proportionality

$$y = ax$$



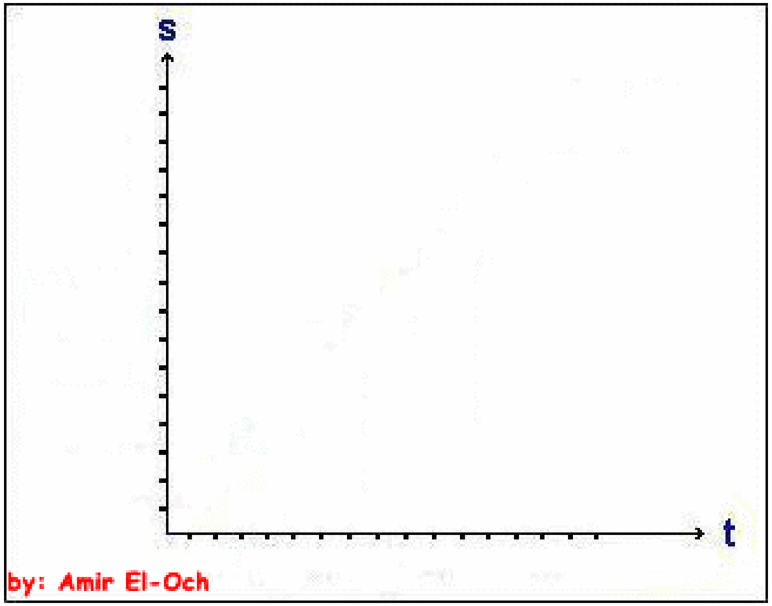
To determine experimentally whether two physical quantities are directly proportional, one performs several measurements and plots the resulting data points in a Cartesian coordinate system. If the points lie on or close to a straight line that passes through the origin (0, 0), then the two variables are probably proportional, with the proportionality constant given by the line's slope.

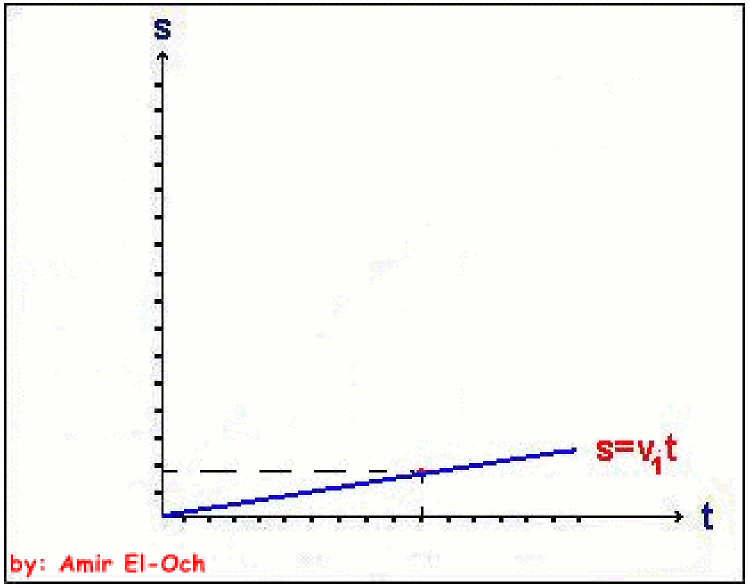
Example: Motion with constant velocity

$\mathrm{v} = \mathrm{const.}$

$$x(t) = s = \mathrm{v} \cdot t$$

- s - dependent variable
- v - slope
- t - independent variable

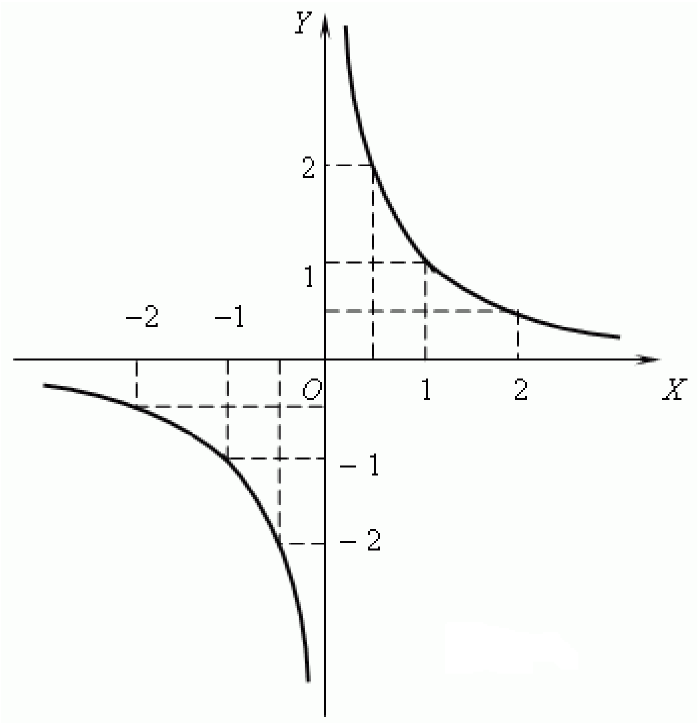




Inverse proportionality

- mathematical form: $$y = \frac{k}{x}$$
- k - constant

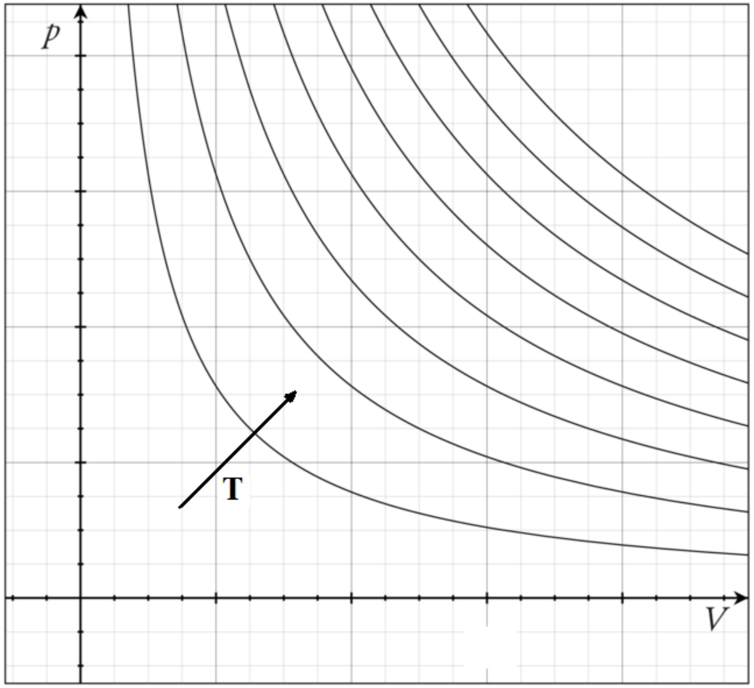
- there are no no zeros of the function; the line representing the function does not intercept the Y-axis



- function is defined for x<0, x>0
- function is not defined for x=0
  - the first-quadrant is only used in physics

Example: inverse proportionality

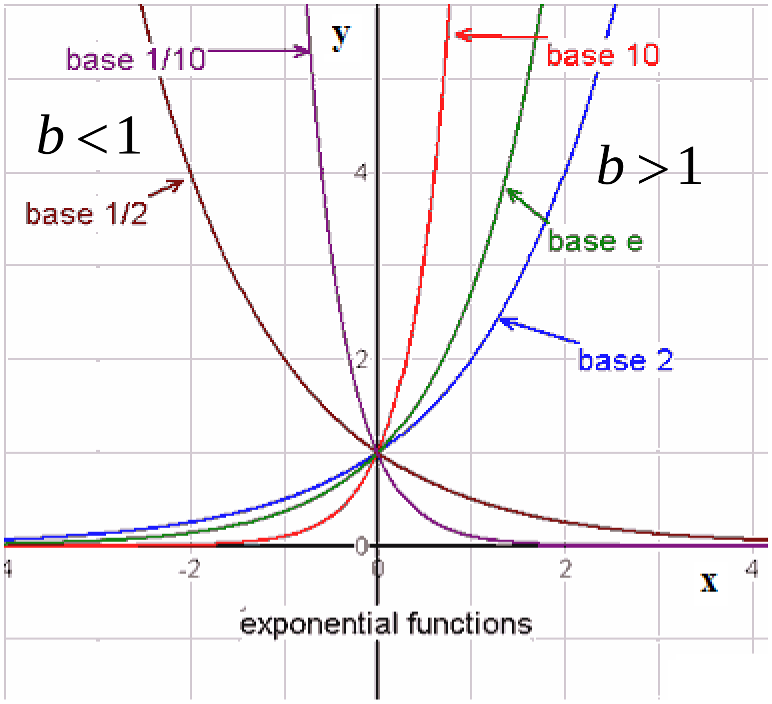
- pressure dependence of the volume for isothermal processes of ideal gas:



$$p = \frac{nRT}{V}$$

Exponential function

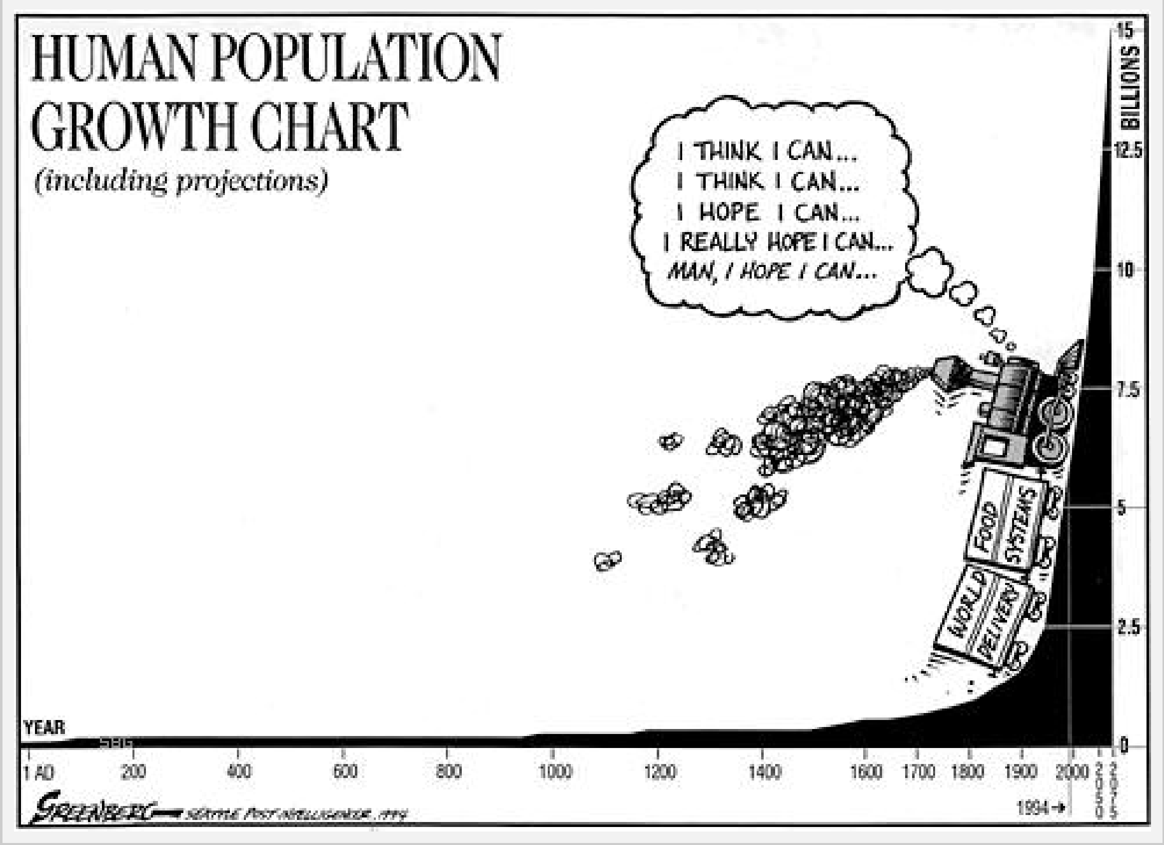
- general form: $y = Aa^{bx}$
- a – base, the most frequently is *e* (the base of natural logarithmic function)
- b – rate changes
- A=y(0) starting value
- there are no zeros of the function
- y>0 !



$$y = a^{bx}$$

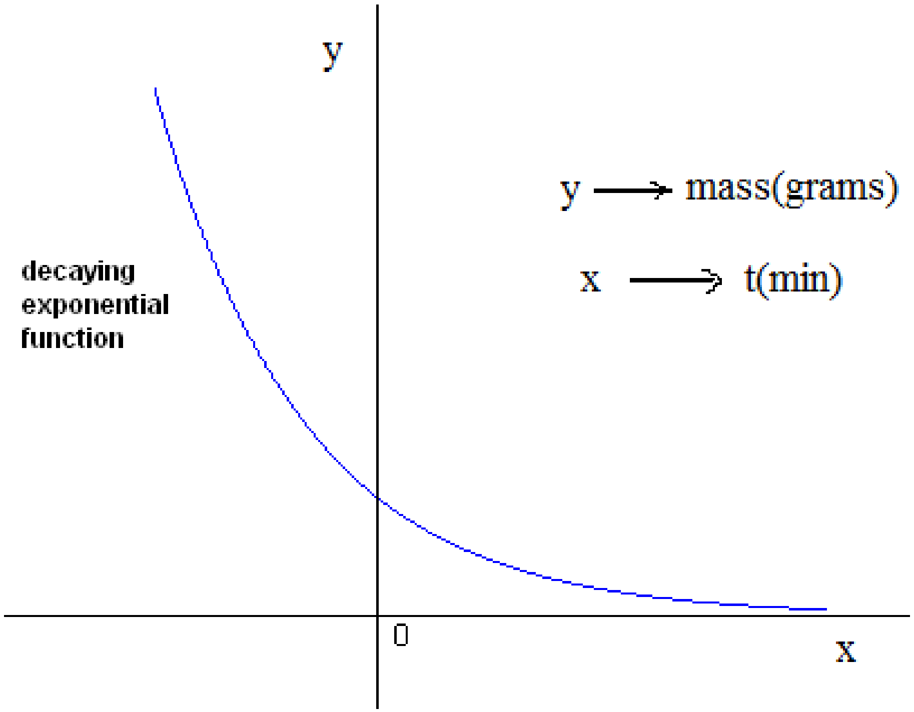
Example: exponential function

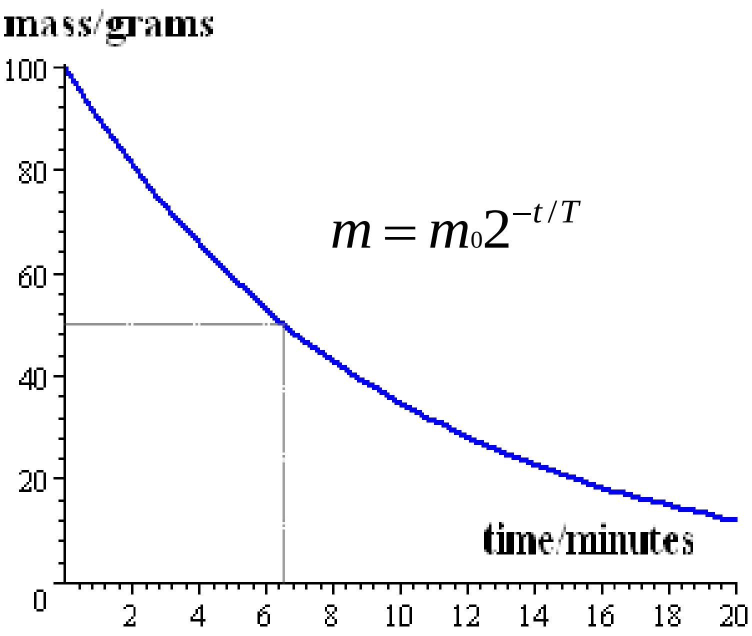
- the most frequent function in description of natural process



- increasing exponential function

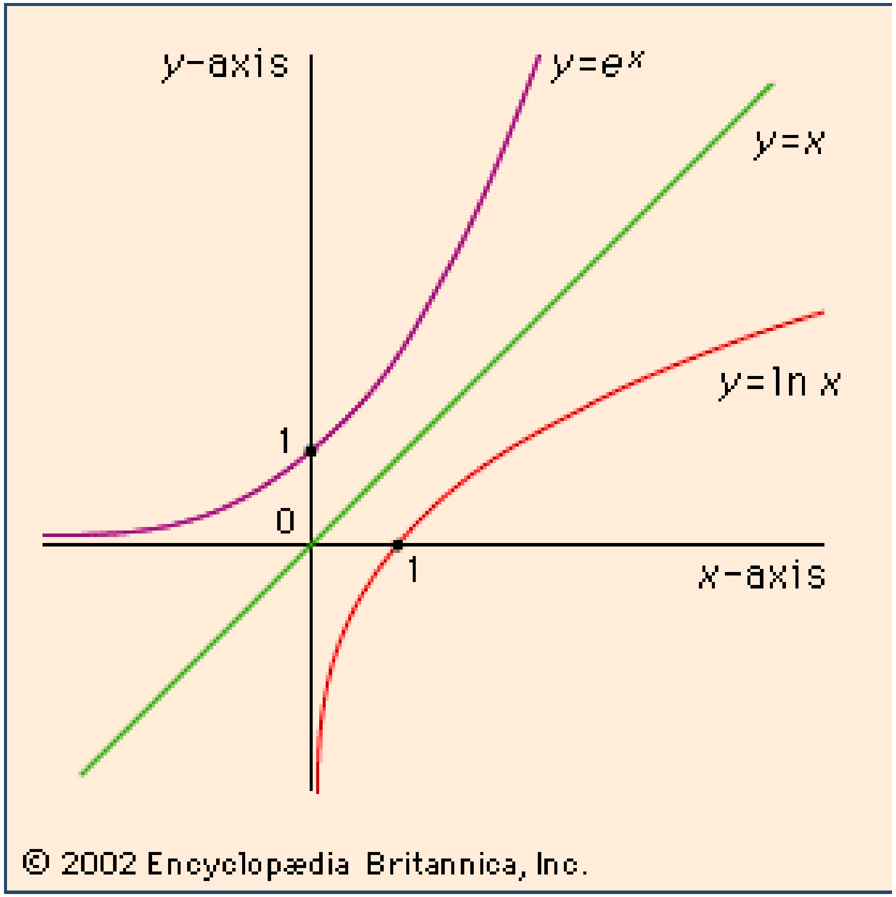
Examples: exponential function





- radioactive decay
- decreasing exponential function

Comparasion



- Logarithmic function increases slower than linear function – helpful for presentation of variables which are changing in great intervals

Periodic functions

- breathing, heart beat, the changes of pressure in the arteries

- the function has the same value after period X:
- $f(x+X) = f(x)$ $f(x+2X) = f(x)$

- time periodicity: $x = \omega t$; $X = \omega T$

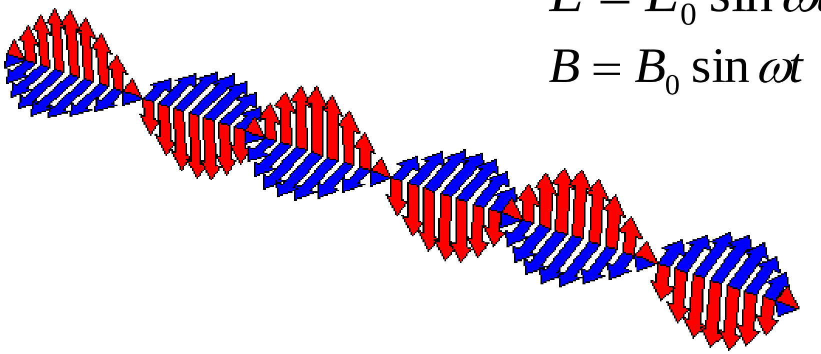
- spatial periodicity: wavelength, $\lambda = \mathrm{v} \cdot \mathrm{T} = \frac{\mathrm{v}}{\mathrm{f}}$

http://en.wikipedia.org/wiki/Periodic_function

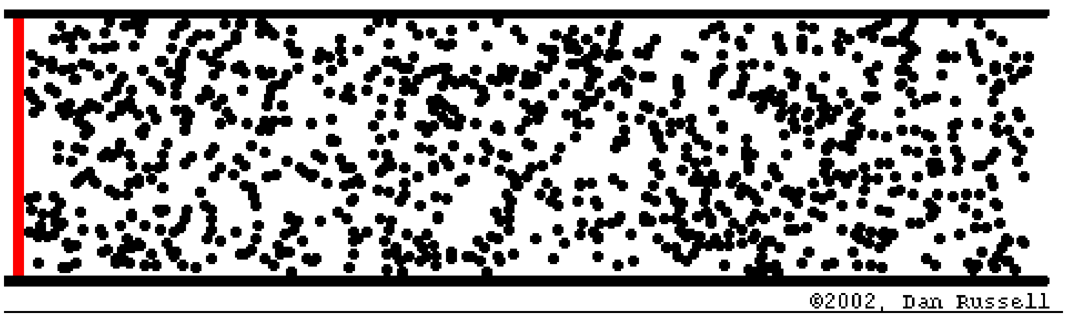
Periodic functions

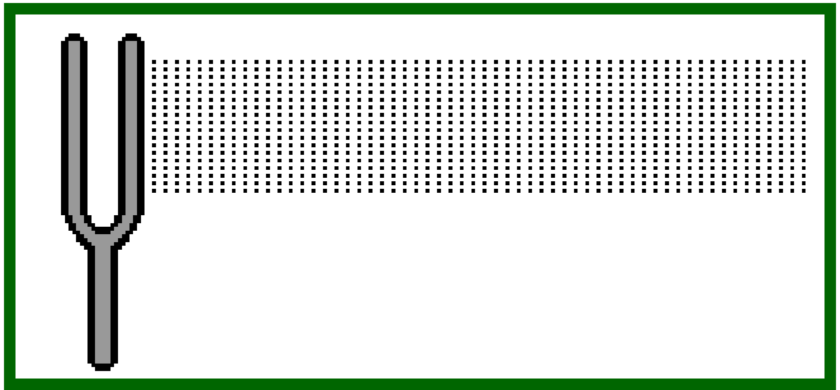
$$E = E_0 \sin \omega t$$

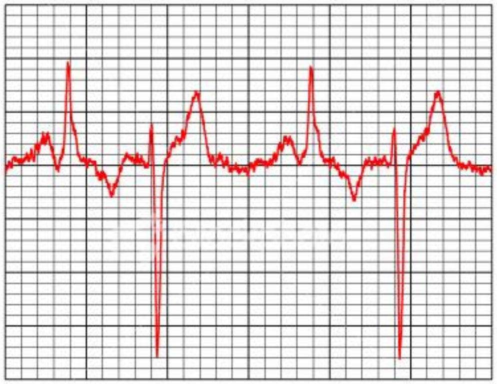
$$B = B_0 \sin \omega t$$



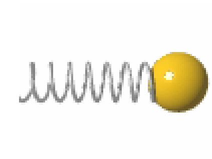
- *space distribution of pressure in sound wave or electric and magnetic fields in electromagnetic wave*







Harmonic function



- the simplest periodic function: free oscillations

http://www.animations.physics.unsw.edu.au/jw/oscillations.htm#Initial

$$y(t) = A\sin\omega t$$

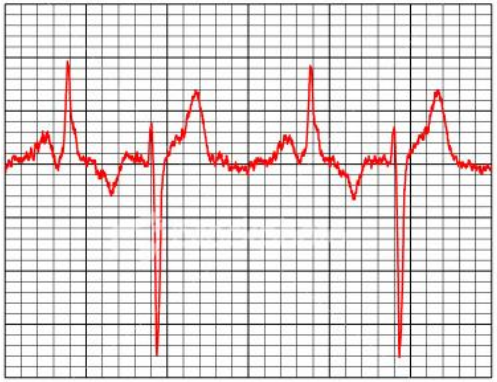
$$\omega = 2\pi f$$

- $y(t)$ – elongation
- A - amplitude
- frequency – the number of oscillation in a 1 second $f = \nu = 1/T$
- general form $y = A\sin(\omega t + \phi)$
- Φ – the phase angle, determined by the starting conditions

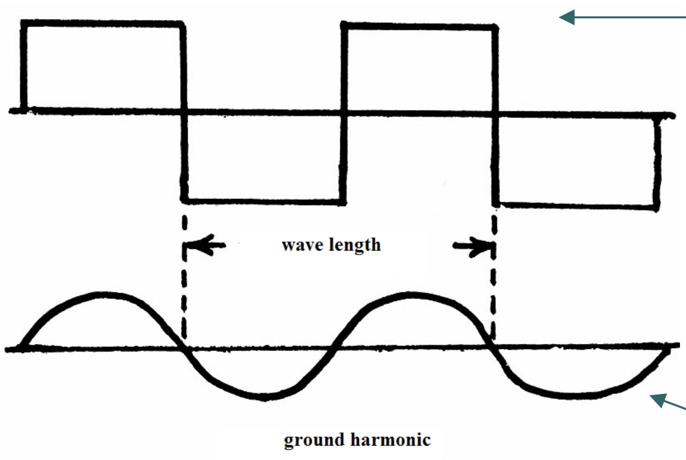
$$y(0) = A\sin\phi$$

Nonharmonic function

- periodic function



- no analitical form

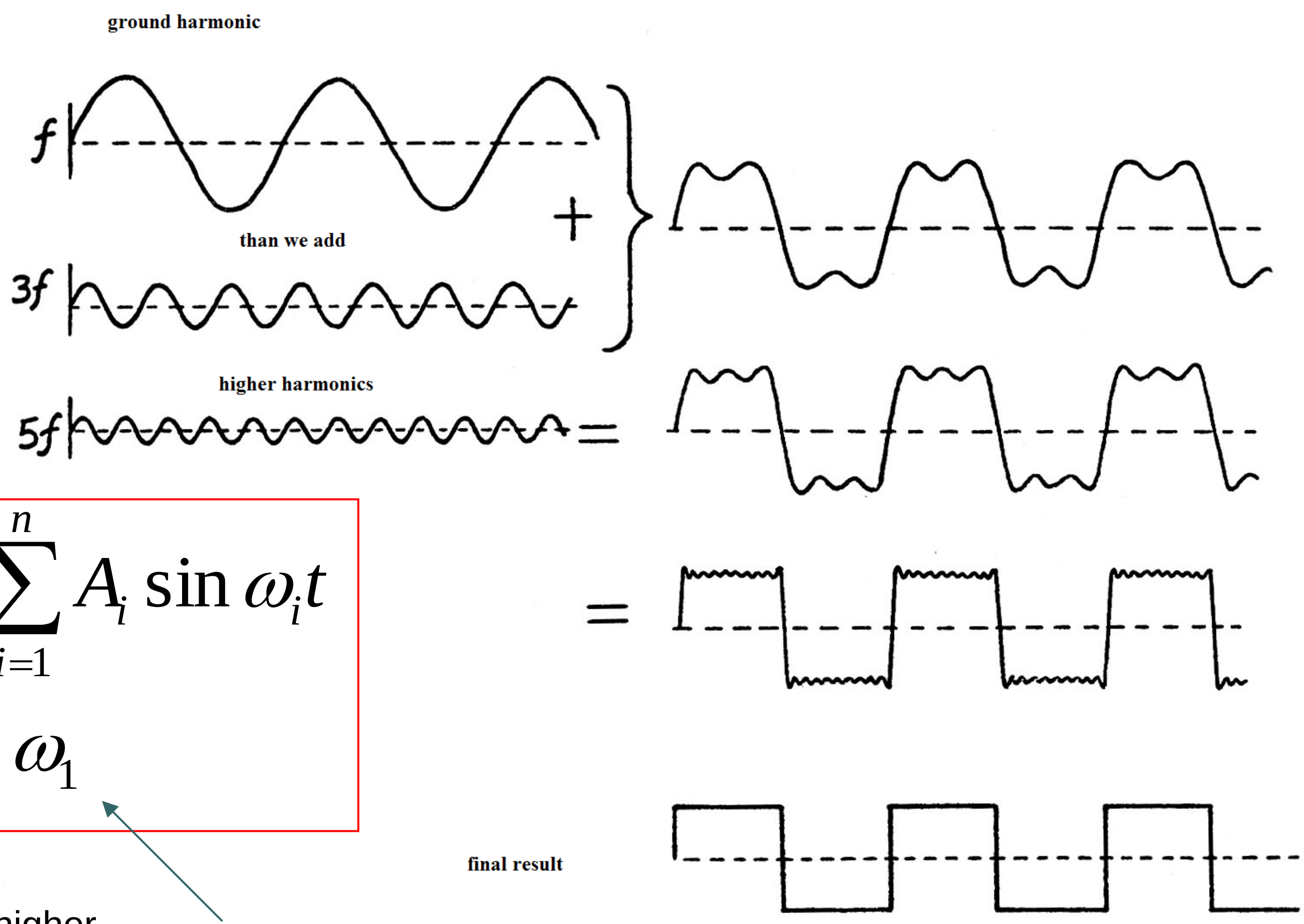


- nonharmonic function
- harmonic function

http://en.wikipedia.org/wiki/Fourier_series

Fourier theorem

- Fourier theorem – nonharmonic function can be represented by the sum of harmonic functions



$$x(t) \approx \sum_{i=1}^{n} A_i \sin \omega_i t$$

$$\omega_i = i \cdot \omega_1$$

- frequency of higher harmonic
- frequency of ground harmonic

Animations:

http://physics.mef.hr/Predavanja/java%20matem%20fje/index.html